Production Network Dynamics and the Propagation of Shocks

Federico Huneeus*

Princeton University

fhuneeus@princeton.edu

Job Market Paper

Download Most Recent Version Here

December 2018

Abstract

This paper uses a firm-to-firm transaction dataset to evaluate quantitatively how shocks propagate through production networks when their underlying links are costly to form and adjust. I document a set of facts consistent with adjustment frictions in these relationships. In particular, these links react sluggishly to firm-specific international trade shocks and are unresponsive to small shocks but strongly responsive to large shocks. Guided by these facts, I develop a dynamic general equilibrium model with endogenous production networks where links have adjustment frictions. Solving for the links' dynamics with a large number of firms is made possible by leveraging the empirical sparsity of firm-to-firm links. To measure the aggregate relevance of these adjustment frictions, I estimate the model using a simulated method of moments and evaluate how international trade shocks during the Great Recession propagated in Chile. Without links' adjustment frictions, and thus with a totally flexible network, the output losses from these shocks would have been 30 percent lower. The application highlights the relevance that dynamics in firm-to-firm links has not only for firms' connectivity but also for how aggregate output responds to shocks.

Keywords: Input-output, networks, trade, firm dynamics, shock propagation.
JEL Classification: D22, D24, D57, F14, F41, L11, L14, L22.

* Department of Economics, Princeton University, Julis Romo Rabinowitz Building, Princeton, NJ 08544. I am extremely grateful to my advisors Richard Rogerson and Esteban Rossi-Hansberg for their continuous support and guidance, and to my committee: Gene Grossman, Atif Mian and Christopher Neilson. I also thank David Baqaee, Adrien Bilal, Jan Eeckhout, Jan De Loecker, Ana Cecilia Fieler, Teresa Fort, Xavier Gabaix, Cristobal Huneeus, Oleg Itskhoki, Gregor Jarosch, Nobuhiro Kiyotaki, Mathilde Le Moigne, Ben Moll, Eduardo Morales, Anders Nielsen, Ezra Oberfield, Fernando Parro, Charly Porcher, Steve Redding and Juan Pablo Xandri, as well as numerous seminar participants for helpful discussions and comments. Raimundo Smith provided superb research assistance. This research benefited from financial support from the International Economics Section (IES), the Simpson Center for the Study of Macroeconomics, the Program for Latin American Studies (PLAS) and the Fellowship of Woodrow Wilson Scholars, all at Princeton University. Their support is gratefully acknowledged. I am also thankful to Claudio Soto and Francisca Perez for providing support for the access to the anonymized data of the Chilean Internal Revenue Service (Servicio de Impuestos Internos or SII). Required disclosure: Information contained here comes from taxpayers' records obtained by the SII, which is collected for tax purposes. This information was used keeping the legal standards of aggregation that prevent the disclosure of private information. The SII assumes no responsibility nor offers guarantees of any kind for the use or application of the aforementioned information in this paper. I thank the SII staff for their support in getting secured access to the anonymized tax records, in particular to Eduardo Pantoja, Pamela Leyton and Conrado Canales, all from the SII's research department.

1 Introduction

Firms form relationships with other firms in order to purchase inputs and to sell their products. Forming these relationships in intermediate input markets is often costly. When asked about challenges that their firms face, managers report that supply-chain disruptions is one of their top concerns (Economist-Intelligence-Unit, 2009).[1] If a firm's supplier receives a shock, and the link is costly to adjust, it might affect the buyer and, indirectly, other firms that are connected to that buyer. Thus, frictions in the adjustment of firm-to-firm links can propagate shocks to other firms. Adjustment frictions in other margins of firms' decisions, such as in labor relationships, final demand, capital markets and price setting, have been studied extensively as channels for the propagation of shocks. Yet, we know little about the relevance of adjustment frictions that characterize firm-to-firm relationships in intermediate input markets. This omission is potentially important, since expenditures on intermediate inputs and sales to other firms account for about half of firms' costs and revenues, respectively.

In this paper, I study how shocks to firms propagate in the economy through production networks in the presence of frictions in adjusting relationships between firms. The aim is to quantify the relevance of the dynamics of adjustment of production links between firms for the propagation of shocks. I combine an administrative business-to-business transaction dataset from Chile with a structural dynamic production network model and show that without adjustment frictions in production links, the international trade shocks during the Great Recession (GR) would have propagated negatively in the Chilean economy by around a 30 percent less.

To implement the analysis, I start by documenting three facts about the dynamics of firm-to-firm links that underlie production networks. First, firms' production links, with both intermediate input buyers and with suppliers, are persistent over time. This persistence increases with firm size. In terms of value, the share of links that are maintained between any pair of years is around 65 percent for the median firm and goes up to around 83 percent for the largest firms.

Second, I use the international trade micro dataset to build firm-specific shocks using a standard shift-share design and evaluate how these shocks propagate through firms' production networks. Domestic production links react to these international shocks, but they take time to adjust. A one standard deviation increase in import prices leads to an increase in the number of domestic suppliers of 5 percentage points after one year, and 10 after 3 years. Thus, the one-year elasticity of changes in import price shocks to the number of domestic suppliers is around half of the 3-year elasticity.

Finally, I find that there is a non-linearity in the response of production links to the size of shocks. Production links are almost unresponsive to small shocks but strongly responsive to large shocks. The increase in the number of domestic suppliers due to import price shocks below the median size of shocks is barely statistically different from zero, whereas the increase for shocks

[1] At the same time, firms spend significant resources in accumulating and keeping customers (Gourio and Rudanko, 2014), and one of those customers are firms. This suggests that forming relationship with customers is also costly.

above the median size is around 40 percent. Furthermore, the difference between the elasticity of small and large shocks is significantly different from zero.

These three facts – persistence, larger long-run than short-run elasticities, and larger elasticities for larger shocks – are consistent with frictions in adjusting firm-to-firm relationships. In order to understand the aggregate relevance of these micro findings, I develop and estimate a model where production links between firms are endogenous and frictional-to-adjust. I evaluate quantitatively how the international trade shocks of the GR propagated domestically in Chile in the context of those frictions.

The setup is a general equilibrium model with heterogeneous firms that can engage in international trade and an endogenous production network in which the formation and adjustment of firm-to-firm relationships is frictional. In the model, firms are exogenously constrained by when they can adjust their links, but conditional on having the opportunity to do so, they endogenously choose which relationships to create and destroy. This friction makes the decision to create and destroy relationships forward looking and thus makes the production network dynamic. This, in turn, implies that the transition path between steady states due to shocks is slow and costly.

The dynamic and forward-looking nature of the decision about which links to create and destroy implies that solving for the full model at a quantitative scale with a large number of firms is a challenging problem. For each potential link, one needs to evaluate the present discounted future bilateral profits, since links cannot be adjusted at every point in time. I deal with this problem by leveraging the fact that the firm-to-firm input-output matrix is sparse. Of the thousands of firms available, the average firm buys from about 30 suppliers and sells to 40 customers. This fact allows me to keep track of a significantly smaller object than the full firm-to-firm input-output matrix.

I exploit this sparsity to simulate and estimate the model with a large number of firms using a simulated method of moments (SMM). This method matches a set of cross-sectional and time series moments of firm characteristics, as well as dynamic patterns of the evolution of firms' production networks. Since the theory includes firms' decisions to engage in international trade, this introduces international shocks to the model, given that Chile is a small and open economy. Firms face demand and productivity shocks in the markets they choose to export to or import from, which allows me to reproduce in the structural estimation the same shift-share design used in the reduced-form analysis. In the spirit of an indirect inference approach, I also target moments from auxiliary regressions, of how the trade shocks propagate indirectly to firms other than the ones directly affected by those shocks. The estimated model is able to reproduce non-targeted features of the data such as the difference between the short-run and long-run elasticities, and the differences between the effects of small and large shocks.

With the estimated model, which provides estimates of the magnitudes of adjustment frictions in intermediate input markets, I evaluate quantitatively how much these frictions contributed to the domestic propagation in Chile of the international trade shocks occurred during the GR. I obtain three main results. First, relative to the initial steady state, the GR trade shocks implied a reduction in output of around 3 percent. Some of this effect comes from the fact that the sluggishness of

production links in the economy makes the effect of the shock longer lived. Second, without these adjustment frictions and thus with endogenous but totally flexible production links, the GR would have implied a decline in output of about 2 percent, or a 30 percent lower than with adjustment frictions. Third, an economy with no non-linearities and efficient allocations would underestimate the effect of the GR by about 47 percent. This suggests that the literature's benchmark, given by Hulten (1978)'s approximation, which is exact under efficient allocations and no non-linearities, is not ideal in this context. These results highlight the fact that taking into account how firm-to-firm relationships adjust and how difficult it is to do so is an important margin when evaluating the propagation of shocks.

This paper connects to three strands of the literature. First, it is related to the recent theoretical literature of endogenous production networks in general equilibrium (e.g., Oberfield (2018), Zou (2018), Taschereau-Dumouchel (2018), Acemoglu and Azar (2017), Tintelnot et al. (2018), Lim (2017), Lu et al. (2017), Eaton et al. (2016), Eaton et al. (2016), Carvalho and Voigtlander (2015), Bernard et al. (2014), Chaney (2014), Atalay et al. (2011)).[2] The theory in this paper is closely related to the work of Lim (2017). who proposed a tractable model of production network dynamics. I extend his framework by including two dimensions that are key for the question this paper addresses. First, I generalize his model to allow for richer substitution patterns in the functional forms of production and demand. This richness is supported by the results of the estimation and is important to take into account non-linearities of how shocks propagate. Second, I extend his model to include international trade, where the original adjustment frictions in domestic firm-to-firm relationships are also present in international linkages, in both export and import activity. This is important in order to replicate in the structural analysis the shocks analyzed in the reduced-form analysis. For a small and open economy like Chile, this is the appropriate setup to evaluate how international shocks propagate domestically. Finally, relative to Lim (2017), this paper contributes by quantifying the relevance of the dynamics of production network for the propagation of shocks which required developing an efficient algorithm to tackle the challenging problem of solving for the dynamics of production links formation in an economy with a rich set of heterogeneous firms.

This paper also relates to the literature on the propagation of shocks through production networks (Tintelnot et al., 2018; Fieler and Harrison, 2018; di Giovanni et al., 2018a,b; Huo et al., 2018; Auer et al., 2018; Kikkawa et al., 2017; Boehm et al., 2017; Carvalho et al., 2016; Barrot and Sauvagnat, 2016; Acemoglu et al., 2016; Caliendo and Parro, 2015; Caliendo et al., 2014; Horvath, 1998; Long and Plosser, 1983). The closest work on this topic is by Tintelnot et al. (2018) and Kikkawa et al. (2017), who also show how international trade shocks propagate using a domestic firm-to-firm transactions dataset but from Belgium. Tintelnot et al. (2018) developed a static model of endogenous production networks and Kikkawa et al. (2017) a model with imperfect competition in firm-to-firm sales. Relative to these papers, I contribute by studying the dynamic response of the production network to international trade shocks. I show direct reduced-form evidence of how

[2]This literature, in turn, is closely related to the one on global supply chains such as Costinot et al. (2013), Alfaro et al. (2015), Fally and Hillberry (2015), Antràs et al. (2017), Antràs and de Gortari (2017).

domestic production links react to international trade shocks. In particular, I show that the links react sluggishly, with a larger long-run elasticity than short-run, and a larger responsiveness to large shocks than to small shocks. I highlight that using a micro, firm-to-firm transaction dataset is key to understanding this question, since data on industry production networks are limited in at least two dimensions. First, industry production networks account for a small fraction of the variation in flows in firms' production networks, even within narrowly defined industry pairs. Second, the extensive margin becomes less important in accounting for the variation in expenditures and revenues in intermediate input markets when viewed at higher levels of aggregation.

Finally, the paper is related to the literature on how adjustment frictions in firms' decision-making matter for the business cycle. It has been a long tradition in macroeconomics to study the impact that adjustment costs in different margins have for firms' output and aggregate output. But although there is significant research on the adjustment frictions in capital (Bachmann et al., 2013), labor (Caballero et al., 1997), final demand (Gourio and Rudanko, 2014) and pricing (Nakamura and Steinsson, 2013), there is little on intermediate input markets.[3] Two exceptions are Heise (2018) and Monarch (2016) who study dynamics in importer-to-exporter relationships. Heise (2018) studies how the length of importer-to-exporter relationships in the U.S. influence the price pass-through of cost shocks in a model with dynamics due to the accumulation of relationship capital. Monarch (2016) estimates the switching costs in importer-to-exporter relationships between firms from the U.S. and China and evaluates the role of those costs for import prices. Relative to this work, I study this problem taking into account the full network behind these firm-to-firm relationships. As mentioned above, solving for the dynamic in these network is challenging if one is to evaluate the aggregate quantitative relevance of adjustment frictions in firm-to-firm links. Adjustment frictions in these markets can be important because these markets account for high shares of firms' costs and revenues, and also because they affect not only the firms that directly face those costs, as with capital or labor adjustment costs, but also the relationship with partner firms, which in turn affects those partner firms indirectly. In addition, the macroeconomic implications of adjustment frictions in these markets can be relevant because they determine the way in which international relationships in global value chains are formed. In the current context of rising connectivity among firms and across countries across the globe due to an increasing dependency on global value chains, understanding the costs of forming international relationships can be important to evaluate the global propagation of shocks and thus the dependence of economies on these global value chains. This paper suggests that any analysis that aims to be quantitatively accurate in evaluating how shocks propagate, whether it is through domestic production networks or through global value chains, should incorporate the dynamics in the formation of these networks. The framework and methods developed in this paper can be used to implement that goal.

The remainder of the paper is organized as follows. The next section presents the main empirical findings. Section 3 introduces the model and Section 4 estimates it. Section 5 develops

[3]The friction in these markets introduced in this paper comes from a dynamic setting, whereas the frictions in intermediate input markets mentioned above in the second set of related literature come mainly from static frameworks.

the quantitative application to the Great Recession and Section 6 concludes. An appendix can be found at the end with details on how the dataset was built, the solution of the model, the algorithm for simulating and estimating the model, and the quantitative application.

2 Firms' Production Network Facts

This section presents the two main observations about firms' production networks dynamics. First, I show that production links are persistence. Second, I use a standard shift-share design of international trade shocks to evaluate how international micro shocks influence the evolution of domestic links and propagate through firms' production networks.

The paper uses three different micro datasets from Chile and one global international trade dataset. First, I use an administrative micro dataset on firm-to-firm transactions. In the data, each firm reports the full list of buyers and suppliers she has, and the flow value of annual firm-to-firm transactions. This information is compiled in order to keep track of value-added tax transactions between firms. Although not all firms have to report this information, they account for around 80 percent of value added in the Chilean economy. The second data has standard firm characteristics such as total sales (which includes sales to final customers, as opposed to sales to intermediate firms), headquarter location and the main industry of the firm.[4] Both datasets are compiled by law by Chile's tax authority, *Servicio de Impuestos de Internos* (SII, for its acronym in Spanish).[5] Third, I use an administrative international trade transaction dataset from Chile's Customs Agency. This dataset has the structure of the state-of-the-art international trade dataset. It includes firms' exports and imports of products to and from all the countries in the world, respectively. Finally, these administrative datasets from Chile are merged with global trade flows between countries of all the products traded in the world. Both the Chilean and this global trade dataset include flows and quantities traded, which allows me to compute unit values of international trade. Appendix F describes all the details of each of these datasets, including how they are built, merged and used in this paper.

2.1 Firms' Production Networks Static and Dynamic Facts

The evidence presented in this section highlights three main ideas: (i) intermediate input markets and their underlying firm-to-firm connections are important for firms in terms of costs and revenues, (ii) the extensive margin accounts for more than half of the variation between firms' costs and revenues from these markets, and (iii) the extensive margin evolution of (ii) is persistent and this persistence increases with firm size.

Fact 1 *Intermediate input markets account for a significant fraction of firms' costs and revenues.* These markets can affect costs because firms buy inputs from suppliers and affect revenues because firms sell their output to other firms. In Chile, intermediate input markets represent on average,

[4]The industry of a multi-industry firm in this dataset would be the industry with the highest revenues for that firm.

[5]Which is the Chilean equivalent to the *Internal Revenue Service*, IRS, in the US.

Table 1: Cross-Sectional Decomposition of Firms' Expenditures and Revenues in Intermediate Input Markets (2011)

	Number of Links	Average		Number of Links	Average
Log. Input Expenditures	0.529***	0.471***	Log. Input Revenues	0.539***	0.461***
	(0.004)	(0.003)		(0.005)	(0.002)
N	18425	18425	N	18425	18425
R^2	0.831	0.762	R^2	0.713	0.721

Notes: OLS regressions of the log of equation (1) implemented for 2011. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

around 52 percent of firms' costs, whereas they represent, around 47 percent of firms' revenues. Thus, these markets are important for understanding firms' production and revenues.

Fact 2 *The number of links account for more than half of the variation in intermediate input transactions across firms, on both the expenditure and the revenue side.* To show this, I apply the standard decomposition from international trade (Bernard et al., 2009) to firms' domestic production networks. Consider firm i's total intermediate input expenditures or revenues at time t, x_{it}, decomposed as

$$x_{it} \quad = \quad n_{it}\bar{x}_{it}, \tag{1}$$

where n_{it} is the number of links (either suppliers or buyers) that firm i has at time t and $\bar{x}_{it} = x_{it}/n_{it}$ is the firm's average expenditure or revenue across its partner firms. Since, in log terms, these components add up to x_{it}, I use ordinary least squares (OLS) to decompose the variation of $\log(x_{it})$. Table 1 presents the results for 2011. It shows that for both expenditures and revenues from intermediate inputs, the extensive margin accounts for around 53 percent of the variation between firms.[6] Similar findings has been found in international trade contexts (Bernard et al., 2018; Bernard and Moxnes, 2018), with similar magnitudes of the relevance of the extensive margin. This fact does not hold, however, when looking at the industry level. Appendix A.1 shows the decomposition at different levels of aggregation of industries. The extensive margin matters more as one goes from aggregate industry-level analysis to more disaggregated industries, to firm-level analysis. This highlights the importance of using micro data at the firm level rather than at the industry level when studying the extensive margin of production networks.

Fact 3 *Production linkages are persistent.* Many of the production links that firms have are persistent over time. The evidence is summarized in Table 2. For the median firm, the fraction of domestic suppliers (buyers) that are retained between two average years is around 41 (46) percent. For links with foreign markets,[7] those numbers are 71 (72) percent, respectively. When weighted by

[6] In Appendix A.1, I show that relatively equal shares of this 53 percent come from the variation in the number of locations and industries firms source from and sell to.

[7] Throughout the paper, domestic firms' links with the rest of the world are measured at the market level, rather than at the firm level, as is done for domestic links. This difference is only due to data constraints. The international trade data do not have information about firm-to-firm trade but rather about firm-to-market trade. Despite this, I use a rather disaggregated measure of international markets, namely country-product combinations, where products

Table 2: Share of Links That Survive Between Pairs of Years - Median Firm - 2011

	Unweighted		Weighted	
	Suppliers	Buyers	Suppliers	Buyers
Domestic	41.3	46.1	64.6	72.1
Foreign	71.5	72.1	75.3	82.2

Notes: This table shows how persistent production links are over time. Evidence on the domestic margin is documented from firm-to-firm relationships, whereas on the foreign margin it is documented from firm-to-market relationships. Foreign markets are defined at the product-country level, whereas products are defined at the 6-digit HS codes classification. The unweighted columns report the shares in terms of number of links. The weighted columns report the share in terms of expenditure (revenue) on suppliers (from buyers).

expenditure or revenues, the magnitude of this persistence is even larger: in the domestic market, the share of expenditure (revenues) that represents links with suppliers (buyers) that are constant between two years are 65 (72) percent. In the foreign market, those numbers are 75 (82) percent. This highlights the fact that the persistence is larger in relationships that involve more flows.

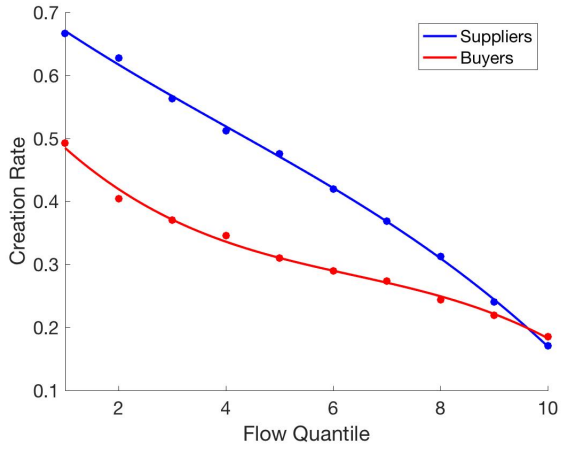
Despite links being measured at a granular level in domestic markets, they are still persistent. Furthermore, although the level of aggregation at which persistence is calculated in domestic and foreign links is different, the persistence is quite similar. The quantitative and aggregate relevance of this fact cannot be evaluated by itself, since it depends on how it is related to other features of firm behavior. This is one of the reasons why the analysis continues with a quantitative model and application to the Great Recession to evaluate the aggregate relevance of this evidence.

Fact 4 *Production links' persistence increases with size.* Larger firms have more persistent relationships. Figure 1 presents the evidence for both domestic and foreign markets, and for suppliers and buyers. These figures report creation and survival rates of firms' production links. Creation rates are the fraction of links at a given moment in time that do not appear the previous year for each firm. Survival rates are the fraction of links at a given time that continue the following year for each firm. The figure shows a significant difference of creation and survival rates of production links between the smallest and largest firms. Larger firms have lower creation rates and higher survival rates. When going from the first to the tenth decile, the share of new domestic links decreases from around 50 and 68, to 20 percent of buyers and suppliers, respectively. Conversely, when going from the first to the tenth decile, the share of links that survive goes from around 40 to 50 percent, for links with both suppliers and buyers. This evidence points to a higher persistence of production links for larger firms.

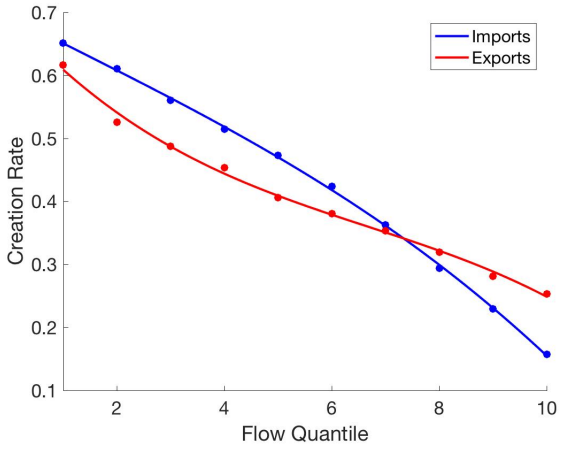
Some of these facts on the persistence of links have been shown previously in the literature. (Lim, 2017) and (Kikkawa et al., 2017) show facts about the dynamics of domestic production networks for the US and Belgium, respectively, whereas (Bernard et al., 2018) show facts about the dynamics of international production networks for Colombia. Quantitatively, the survival rates

are defined at the 6-digit level of the harmonized system (HS) classification. Both the methods for measuring links' persistence and also the theory developed in Section 3 are not specific to the level of aggregation and can be easily extended to allow for firm-to-firm links in international trade. Those extensions are left for future research.

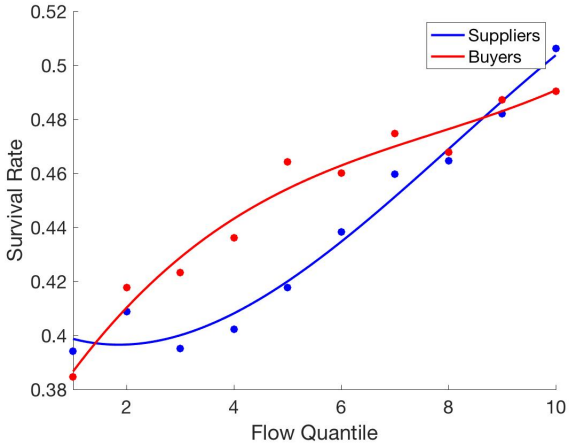
Figure 1: Creation and Survival Rates and Size - Unweighted - 2006



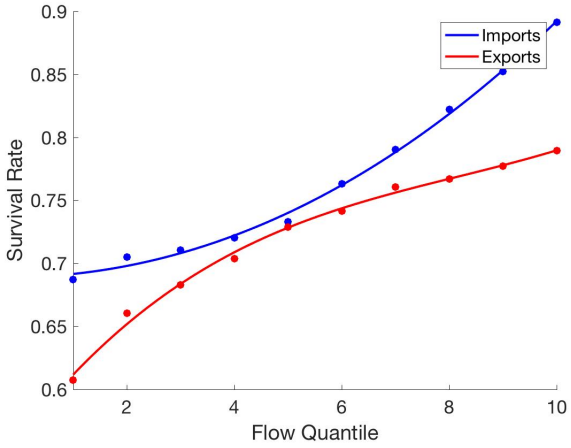
(a) Creation: Domestic Links



(b) Creation: Foreign Links



(c) Survival: Domestic Links



(d) Survival: Foreign Links

Notes: These figures document creation and survival rates of intermediate input links, in both domestic and foreign markets, as well as on the buyer and supplier side. Creation rates are the fraction of links at a given moment in time that do not appear the previous year for each firm. Survival rates are the fraction of links at a given time that continue the following year for each firm. The measure of size in each graph is the total flows of each firm in the corresponding margin. For example, the size measure of the creation rate of domestic suppliers is the total expenditure in domestic suppliers of each firm. Similar graphs can be obtained using different measures of size such as total sales. The problem with this variable is that it is not observed across the whole distribution because a subset of firms export or import from international markets. Links in this evidence are not weighted and they are documented for 2006.

of domestic production links in Chile are smaller than the ones in the US and Belgium in terms of counts but similar in terms of value.[8] Relative to international production links, although the

[8]It is important to note that the data used in Lim (2017) for the US is only for publicly listed firms and among those, only the main links that those firms have. Given that persistence increases with size, it is possible that the facts for the US are overestimating the persistence of the links in the overall economy.

dataset used by Bernard et al. (2018) for documenting persistence in Colombia is of a different nature than the one I use for Chile, they also find evidence for some persistence in international production links.

2.2 Direct Evidence on the Propagation of Micro Shocks

I provide direct evidence on how micro shocks propagate through production networks. In particular, I show two features: (i) international trade micro shocks propagate through production networks, affecting indirectly firms that are not directly affected by those shocks and (ii) the number of production links react to these shocks, but sluggishly. Moreover, the links are unresponsive to small shocks but strongly responsive to large shocks.

Before showing the evidence, I describe the micro shocks that are used for this exercise. As is standard in the international trade literature, I use a shift-share design at the firm level in the spirit of Hummels et al. (2014) and Bernard et al. (2018).[9] I consider import price shocks, denoted by $\Delta \log p^I_{i,t}$ for firm i in year t, and export demand shocks, denoted by $\Delta \log d^E_{i,t}$. These are defined as

$$\Delta \log p^I_{i,t} = \sum_{k \in \Omega^I_{i,0}} s^I_{ik,0} \Delta \log p^G_{k,t}, \tag{2}$$

$$\Delta \log d^E_{i,t} = \sum_{k \in \Omega^E_{i,0}} s^E_{ik,0} \Delta \log d^G_{k,t}, \tag{3}$$

where $\Omega^I_{i,0}$ ($\Omega^E_{i,0}$) is the set of markets k that firm i sources (sells) from (to) in $t = 0$, $s^I_{ik,0}$ ($s^E_{ik,0}$) is the share of imports (exports) of firm i from (to) market k in $t = 0$ and $\Delta \log p^G_{k,t}$ is the percentage growth of the export price of market k to the rest of the world, excluding Chile. Similarly, $\Delta \log d^G_{k,t}$ is the percentage growth of imports of market k from the rest of the world, excluding Chile.[10] In short, the shocks represent percentage changes in the price (flow) of imports (exports) that Chilean firms would face if they took the rest of the world average price (flow) as given, averaging across markets according to the relevance of these markets for them.[11]

Since Chile is a small and open economy, Chilean firms can take changes in international markets as exogenous. Nevertheless, since firms endogenously source from foreign markets and use information about prices and demand in these markets to make those decisions, the size and openness of the Chilean economy are not sufficient to guarantee exogeneity. The analysis here follows the shift-share design taken in Adão et al. (2018) by assuming exogeneity from the shifts,

[9] These shocks can be thought of as firm-level versions of the empirical strategy in Autor et al. (2013).

[10] Bernard et al. (2018) argue that it is preferable to use shares instead of flows for these shift-share shocks in order to control for aggregate demand shocks, as opposed to Hummels et al. (2014) who use flows. This concern is addressed by including year fixed effects in the regressions.

[11] The theory in Section 3 justifies why one should use price shocks on imports and demand shocks on exports. The intuition is that in the model prices are set according to the seller's marginal cost. This implies that import prices are defined internationally. This together with the fact that Chile is a small economy, means that Chilean firms take the foreign marginal cost and import prices as given. For this same reason, export prices are determined by domestic firms' marginal cost, so they cannot be a good measure of foreign shocks. Thus, I use export demand shocks, since those are set internationally and domestic firms take those as given.

rather than from the shares.[12] As will be clear when the theory is introduced, the logic behind the shift-share shocks is that firms choose which market to connect to (either through imports or exports) and are subject to the shocks that those markets face. Thus, under a standard international trade setup where firms choose these markets, the natural strategy for the design is to treat the shares as endogenous and the shifts as exogenous. Nevertheless, when firms make the decision about which markets to link to, future shocks to those markets could already be in firms' information set. The usual strategy to avoid this concern is to use lagged shares of exposure to those markets. I follow this strategy and complement it with another piece of evidence: current shares are not significantly correlated with future shocks in both import and export markets. Appendix A.2 presents this evidence that supports the exogeneity of these shocks.

One of the advantages of using shift-share shocks is their heterogeneity across firms. This heterogeneity comes from two sources. First, from firms' decision about which international markets to source from or sell to, that is, from the extensive margin of international trade. Although all firms are potentially affected by every global market, in practice the exposure is heterogeneous. In the data, there are, in practice, around 450,000 markets, which are defined by product-country combinations where products are defined at the 6-digit HS code level. This makes it unlikely that any two firms are exposed to the same markets. In fact, the median number of Chilean firms that import (export) from (to) the same market is two (one). This is in part given by the fact that the median number of markets firms import or (export) to (from) is five (five) and that there are around 450,000 markets available. The second source of heterogeneity comes from firms' decision about how much to import from or export to, given a set of markets with which they are connected to. Even though any two firms might be connected to the same markets, the intensity of their connection is heterogeneous. To evaluate this, I measure how much of the variation in firms' trade flows is accounted for by market-year fixed effects. In other words, fixing the set of markets two firms have, I measure how much the intensive margin accounts for the total variation in trade flows with those markets. I find that in import (export) activity, market-year fixed effects account for around 37 (43) percent of the variation in trade flows across firms. Evidently, the extensive margin accounts for the majority of trade flows, but the intensive margin is still important. Finally, to complement the evidence of the heterogeneity of these shocks across firms, I test how much of the variation in the final shock variables $\Delta \log p^I_{i,t}$ and $\Delta \log d^E_{i,t}$ is accounted for by industry-time variation as opposed to firm-time variation. Around 24 (27) percent of the variation in the export (import) shocks is accounted for by industry variation.[13] All of this evidence supports the conclusion that the shocks are significantly heterogeneous across firms and justifies measuring the shocks at the firm level.

I measure how micro shocks propagate through the economy using the following specification[14]

[12] This strategy is opposite to the one used in Goldsmith-Pinkham et al. (2018). They argue for exogeneity through the shares, rather than the shifts.

[13] This is done at the equivalent of 6-digit industries, representing around 650 industries.

[14] These specifications have some similarities with some specifications of earlier versions of Tintelnot et al. (2018) and Kikkawa et al. (2017). The main difference is the definition of the shocks. Tintelnot et al. (2018) use international trade flow shocks for both imports and exports and Kikkawa et al. (2017) use import trade flows from China, whereas

Table 3: Propagation of International Trade Shocks

	Δ Log Sales		Δ Log Number Buyers		Δ Log Number of Sellers	
	(1)	(2)	(3)	(4)	(5)	(6)
Direct Import Shock	-0.768***	-0.753***	0.232***	0.211***	-0.011	-0.016
	[0.121]	[0.101]	[0.079]	[0.062]	[0.035]	[0.011]
Upstream Import Shock	-0.830**	-0.630***	0.631***	0.520***	0.130*	0.125
	[0.415]	[0.215]	[0.052]	[0.034]	[0.079]	[0.091]
Downstream Import Shock	0.857**	0.857**	-0.051	-0.022	-0.121***	-0.101***
	[0.429]	[0.429]	[0.075]	[0.031]	[0.041]	[0.023]
Direct Export Shock	0.318***	0.311***	0.507***	0.314***	-0.418***	-0.323***
	[0.105]	[0.118]	[0.174]	[0.112]	[0.101]	[0.092]
Upstream Export Shock	-0.513	-0.062	-0.032	-0.011	0.110	0.103
	[0.719]	[0.077]	[0.041]	[0.021]	[0.312]	[0.253]
Downstream Export Shock	0.399***	0.312***	0.102***	0.082***	0.031	0.010
	[0.084]	[0.096]	[0.041]	[0.028]	[0.027]	[0.014]
N	14321	14321	13141	13141	12012	12012
R^2	0.971	0.970	0.709	0.652	0.742	0.661
Industry FE		✓		✓		✓
Mean DV	15.07	15.07	0.42	0.42	0.51	0.51
SD DV	2.02	2.02	1.02	1.02	1.13	1.13

Notes: Results of OLS regressions of Equation (4). Implemented on differences between 2011 and 2006. Each regression also has a set of controls. In particular, each regression includes a dummy for import and export activity, as well as dummies for import and export activity of partner firms. These dummies are defined depending on whether the upstream and downstream shocks are non-zero. Standard errors in parentheses, double clustered at the firm and year level. *** p<0.01, ** p<0.05, * p<0.1.

$$\begin{aligned}\Delta \log y_{i,t} &= \alpha_1 \Delta \log p_{i,t}^{I,O} + \alpha_2 \Delta \log p_{i,t}^{I,U} + \alpha_3 \Delta \log p_{i,t}^{I,D} + \alpha_4 \Delta \log d_{i,t}^{E,O} + \alpha_5 \Delta \log d_{i,t}^{E,U} \\ &\quad + \alpha_6 \Delta \log d_{i,t}^{E,D} + \mathbf{X}_{i,t}\beta + \gamma_i + \delta_t + \varepsilon_{i,t},\end{aligned} \tag{4}$$

where $\Delta \log y_{i,t}$ is the log change in outcome y for firm i in year t, $\Delta \log p_{i,t}^{I,O}$ is the log change in the import price shock that firm i receives directly, $\Delta \log p_{i,t}^{I,U}$ and $\Delta \log p_{i,t}^{I,D}$ are the log change in the import price shock that suppliers and buyers in upstream and downstream links receive, respectively. The variables for the export flow shock are similar. Finally, $\mathbf{X}_{i,t}$ is a list of controls, γ_i is a firm fixed effect, δ_i is a year fixed effect and $\varepsilon_{i,t}$ is an exogenous non-systematic residual. The results of this analysis are in Table 3.

Fact 5 *International Trade Shocks Propagate Through Firms' Production Networks.* Table 3 shows that firms exposed directly to higher import price shocks see their revenues decrease. Also, firms exposed directly to higher export flow shocks see their revenue increase. In terms of magnitudes, an increase of one standard deviation on a direct import (export) price (flow) shock decreases (increases) sales by 0.7 (0.3) percent. This is a natural result that would appear from a basic model where firms receive cost (demand) shocks. What is more important is that shocks that a firm's suppliers and buyers receive also affect its revenues. Productivity shocks propagate

I use price shocks on imports and flow shocks on exports. The rationale for this decision comes from the structure of the model that I use to analyze these specifications in section 4.

downstream and demand shocks propagate upstream. If a firm has a supplier that enjoys a positive shock to its import prices, the firm sees its revenues decrease. This result is consistent with a positive pass-through of the cost shock from the supplier to the buyer, which would imply an increase in the buyer's price and thus a reduction in its revenues. On the export side, there is no evidence for the downstream propagation of upstream export shocks. Finally, if a firm has a buyer that faces a positive export shock, it will see its revenues increase. This is consistent with a standard mechanism where higher downstream demand increases the demand for upstream inputs.

Table 3 also shows that domestic production links react to international trade shocks. In particular, firms increase their numbers of suppliers when they experience a positive export shock or a negative import shock. This is consistent with standard firm models where positive demand shocks allow for the expansion of production and thus inputs, and negative import shocks, i.e., increase in import prices, induce substitution toward domestic varieties.[15] To understand how difficult it is for firms to adjust their links, I evaluate the timing of these changes by using the following specification

$$\Delta_h \log n^S_{i,t+h-1} = \sum_{k=1}^{3} \alpha_k^{I,h} \Delta \log p^I_{i,t-k} + \sum_{k=1}^{3} \alpha_k^{E,h} \Delta \log d^E_{i,t-k} + \mathbf{X}_{i,t}\beta + \alpha_i^h + \gamma_t + \epsilon^h_{i,t+h-1}, \qquad (5)$$

where $\Delta_h \log n^S_{i,t+h-1}$ are log changes in the number of links over a horizon of h years for firm i at year $t+h-1$, α_i^h are firm i's fixed effects, $\Delta \log p^I_{i,t-k}$ ($\Delta \log d^E_{i,t-k}$) are the import (export) price (flow) shocks defined in equation (2) ((3)) that firm i faces in $t-k$, $\mathbf{X}_{i,t}$ is a set of covariates including export and import status dummies, γ_t are year fixed effects and $\epsilon^h_{i,t+h-1}$ are residuals at the firm i, year $t+h-1$ level. This specification is standard in the literature and has been found to be robust to misspecifications (Jordà, 2005).

Fact 6 *Production Links React Sluggishly to Shocks.* Figure 3 presents estimates of $\alpha_1^{I,h}$. It shows that the short-run and long-run elasticities of the number of domestic links to import price shocks is significantly different. The three-year elasticity is around twice the one-year elasticity. An increase of one standard deviation in import price shocks implies an increase of ten percent in their number of suppliers after three years.

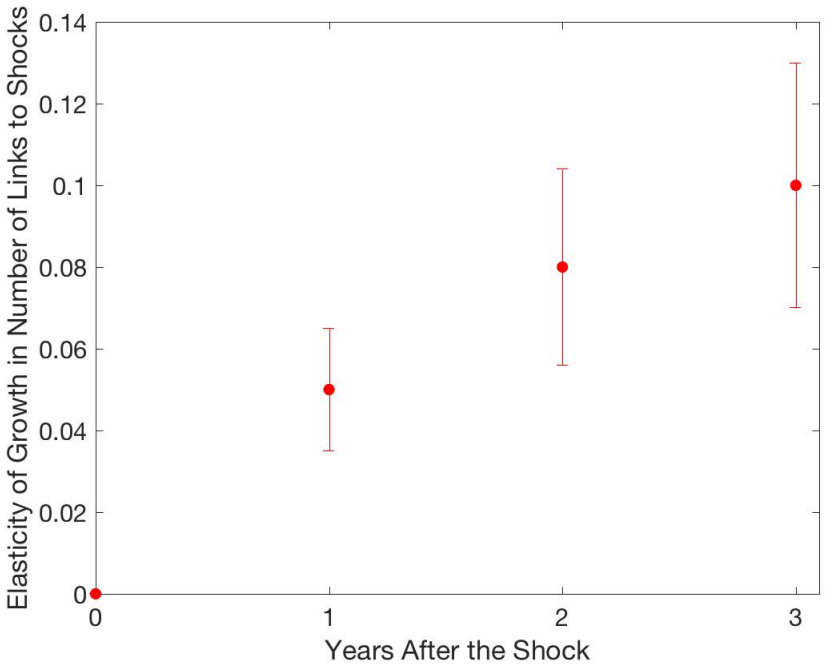
Fact 6 is consistent with links being sticky and costly to adjust over time. In order to further test this hypothesis, I evaluate whether links react differently to small versus large shocks. To do this, I run the following specification

$$\Delta \log n^S_{i,t} = \alpha_1 \Delta \log p^{I,s}_{i,t} + \alpha_2 \Delta \log p^{I,l}_{i,t} + \alpha_3 \Delta \log d^{E,s}_{i,t} + \alpha_4 \Delta \log d^{E,l}_{i,t} + \gamma_i + \delta_t + \varepsilon_{i,t}, \qquad (6)$$

where $\Delta \log n^S_{i,t}$ is the log change in the number of suppliers firm i has at time t, $\Delta \log p^{I,s}_{i,t}$ is the import price shock defined in equation (2) of shocks below the median of the size distribution of the

[15] The effects on the number of sellers are weaker and thus I avoid its discussion here. Furthermore, the result in Columns 5 and 6 that a positive direct export shock reduces the number of domestic sellers is not consistent with a basic model with constant returns to scale technology, since demand decisions would be independent of each other. The model in this paper will not be able to replicate this finding. An extension with increasing returns to scale might be able to reproduce this finding and may be an interesting avenue for future research.

Figure 3: Sluggishness of Links' Reaction to Import Price Shocks



Notes: This figure shows estimates of coefficients $\{\alpha_1^{I,h}\}_{h=1}^3$ of Equation (5) together with confidence intervals of 95 percent.

absolute value of the shock.[16] Similarly, $\Delta \log p_{i,t}^{I,l}$ are the import shocks of shocks above the median of the size distribution of the absolute value of the shock. The same logic applies to export flow shocks in $\Delta \log d_{i,t}^{E,s}$ and $\Delta \log d_{i,t}^{E,l}$. Finally, γ_i and δ_t are firm and year fixed effects, respectively, and $\varepsilon_{i,t}$ is a residual.

Fact 7 *The Number of Links Are Unresponsive to Small International Trade Shocks, but Strongly Responsive to Large Shocks.* Table 4 shows that, as a reaction to international import prices and export demand shocks, the number of buyers is almost unresponsive to small shocks but reacts significantly to large shocks. In terms of magnitudes, relative to import price (export demand) shocks that are above the median of the absolute size of these shocks, a one standard deviation increase in this price increases the number of buyers by around 45 (64) percent. These results are robust to controlling for industry-year fixed effects and highlight the fact that the response of domestic links to international shocks is non-linear in the size of the shock.

The facts presented so far can be summarized in four main messages:

1. Firm-to-firm links are important for firms.

2. These links are persistent over time and this persistence increases with firm size.

3. Micro international trade shocks propagate through firms' production networks.

[16] This means that, for example, $\Delta \log p_{i,t}^{I,s} = \Delta \log p_{i,t}^{I}$ if the absolute value of $\Delta \log p_{i,t}^{I}$ is smaller than the median of the absolute value of $\Delta \log p_{i,t}^{I}$, and $\Delta \log p_{i,t}^{I,s} = 0$ otherwise.

Table 4: The Reaction to Small versus Large International Trade Shocks

	Δ Log Number Buyers	
	(1)	(2)
Small Import Price Shock	0.159*	0.121
	[0.091]	[0.112]
Large Import Price Shock	0.446**	0.454***
	[0.191]	[0.124]
Small Export Demand Shock	0.017*	0.017*
	[0.010]	[0.010]
Large Export Demand Shock	0.637***	0.414***
	[0.124]	[0.106]
N	13201	13201
R^2	0.708	0.652
Industry FE		✓
Mean DV	0.44	0.44
SD DV	1.03	1.03

Notes: This table shows the OLS estimates of equation (6). Implemented on differences between 2011 and 2006. Large and small shocks are defined as shocks above and below the median of the size distribution of the shock, respectively. *** p<0.01, ** p<0.05, * p<0.1.

4. Production links are endogenous to international shocks and react sluggishly and non-linearly to the size of the shocks.

These facts can be rationalized with production links being sticky in intermediate input markets and thus with frictions in adjusting them. Also, they suggest that as firms get larger, they establish relationships that are more difficult to adjust. Thus, when a shock hits a link of these large firms, they might be unable to adjust that link, which in turn might amplify the effect of the shock on other firms. If this mechanism works more strongly in larger firms, then one could anticipate that it might be consequential for the aggregate economy. Another reason why these frictions might be relevant in the aggregate is because intermediate input markets contribute a large fraction of firms' costs and revenues. In order to understand quantitatively the aggregate consequences of these frictions for the propagation of micro shocks, I propose a dynamic theory of production networks.

3 A Dynamic Model of Production Networks

This section presents a general equilibrium model of production networks that features frictions in the adjustment of network links. The purpose of the theory is to shed light no how such frictions affect the propagation of micro shocks.[17] This section presents the open economy version of the model. Although some of the main forces also work in a closed economy, introducing the open economy is important for the quantitative application to the Great Recession in Section 5.

[17]Throughout the paper, micro shocks, shocks to firms' primitives, and idiosyncratic shocks are concepts used interchangeably.

3.1 Production Network Dynamics in an Open Economy

I begin by describing a model that takes production networks' links as given.[18] Then I allow for the extensive margin of the production network to be endogenous and subject to frictions, thereby introducing dynamics. The model is an extension to the one in Lim (2017). It generalizes his model to include a richer structure of substitution, in both demand and production, and to incorporate international trade. These features allow for a better quantification of the role of adjustment frictions in production networks for the propagation of international micro shocks. All proofs are in Appendix B.4.

3.1.1 Exogenous Production Network in an Open Economy

The economy comprises two asymmetric countries, denoted by Home and Foreign (H and F), respectively, and indexed by i. Each country is populated by a representative household and an exogenously given unit continuum of firms indexed by $j_i \in [0,1]$.[19] Each firm produces a unique variety of a differentiated good. Firms have different productivities and demand qualities. The characteristics of firm j_H are denoted by $\phi_H(j_H) = (\phi_H^C(j_H), \phi_{FH}^P(j_H), \phi_H^L(j_H))$, where $\phi_H^C(j_H)$ is a demand shifter for quality coming from the Home household's consumption, $\phi_{FH}^P(j_H)$ is a productivity shifter in the production in the Foreign country when using varieties from the Home country;[20] and $\phi_H^L(j_H)$ is a labor-augmenting productivity term.[21] The characteristics of firm j_F are denoted by $\phi_F(j_F) = (\phi_F^C(j_F), \phi_{HF}^P(j_F))$, where $\phi_F^C(j_F)$ is a demand shifter for quality coming from the Foreign household's consumption, and $\phi_{HF}^P(j_F)$ is a productivity shifter in the production of Foreign varieties that Home firms may import. Firms are characterized by ϕ_i, which I denote as types.[22] There is an exogenous probability function over firm types denoted by G_{ϕ_i}, with density g_{ϕ_i} and bounded support $S_{\phi_H} \in \mathbb{R}_+^3$ and $S_{\phi_F} \in \mathbb{R}_+^2$.[23] Finally, international trade, through the purchase of imported intermediate inputs and the selling of exports, is subject to a standard iceberg trade cost $\tau_F > 1$.

Each firm produces output by combining labor and intermediate inputs provided by suppliers. In order to use intermediate inputs produced by others, a firm must establish relationships in intermediate input markets, thereby creating a production network between firms. These relationships are formed subject to frictions. Every domestic type-ϕ_H firm is only able to purchase inputs from another domestic type-ϕ'_H firm's output with probability $m_H(\phi_H, \phi'_H)$, i.i.d. across potential re-

[18] Throughout the paper, production network's extensive margin, production links, production relationships, and firm-to-firm links are concepts used interchangeably.

[19] For the static presentation of the model, time subscripts are avoided to simplify the exposition.

[20] This could also be interpreted, from the perspective of the exporter, as an international demand shifter for quality.

[21] Using labor-augmenting instead of Hicks-neutral productivity factors shows theoretically more clearly how the production network interacts with firms' productivity. It also induces a more conservative analysis of the propagation of micro shocks as argued in Baqaee and Farhi (2018a).

[22] In the solution of the model, for each type ϕ_i there is a continuum of firms j_i that make identical decisions. Thus, j_i indices can be avoided.

[23] Since there is neither entry nor exit of firms, all integrands over firm types cover the full support of G_{ϕ_i}. Thus, for simplicity of notation, its reference is avoided in such integrands.

lationships. Similarly, every domestic type-ϕ_H firm is only able to purchase inputs from another foreign type-ϕ_F firm's output with probability $m_F(\phi_H, \phi_F)$, i.i.d. across potential relationships. I refer to the functions $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$ as the *matching functions*. Given that there is a continuum of firms for every type, $m_H(\phi_H, \phi'_H)$ is also equal to the fraction of type-ϕ_H firms that purchase from a given type-ϕ'_H firm, as well as the fraction of type-ϕ'_H firms that supply a given type-ϕ'_H firm. These functions characterize entirely the extensive margin of the production network in the economy.

Given $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$, the output of a firm ϕ_H from Home is given by a nested constant elasticity of substitution (CES) and constant returns to scale (CRS) production function over labor and intermediate inputs,

$$y(\phi_H) = \left[\alpha^L \left(\phi_H^L l(\phi_H)\right)^{\frac{\sigma^L-1}{\sigma^L}} + (1-\alpha^L) x^T(\phi_H)^{\frac{\sigma^L-1}{\sigma^L}}\right]^{\frac{\sigma^L}{\sigma^L-1}}, \tag{7}$$

where $l(\phi_H)$ is the quantity of labor demanded by a firm of type ϕ_H, $x^T(\phi_H)$ is the composite of intermediate goods demanded, α^L represents the labor intensity of the economy, and σ^L is the elasticity of substitution between labor and intermediate inputs. The total intermediate input usage by a firm of type ϕ_H, is a CES aggregate of domestic and international intermediate inputs:

$$x^T(\phi_H) = \left[x_H^G(\phi_H)^{\frac{\sigma^X-1}{\sigma^X}} + \alpha^I x_F^G(\phi_H)^{\frac{\sigma^X-1}{\sigma^X}}\right]^{\frac{\sigma^X}{\sigma^X-1}}, \tag{8}$$

where $x_H^G(\phi_H)$ and $x_F^G(\phi_H)$ are the quantities of intermediate inputs demanded in Home and Foreign, α^I represents the import intensity in the economy, and σ^X is the elasticity of substitution between home and foreign varieties of intermediate inputs. Finally, firm ϕ_H's demand for intermediate inputs from Home and Foreign is also CES aggregators over different varieties, namely

$$x_i^G(\phi_H) = \left[\int m_i(\phi_H, \phi'_i) x_i^P(\phi_H, \phi'_i)^{\frac{\sigma_{Hi}^P-1}{\sigma_{Hi}^P}} dG_{\phi_i}(\phi'_i)\right]^{\frac{\sigma_{Hi}^P}{\sigma_{Hi}^P-1}}, \quad i=\{H,F\}, \tag{9}$$

where $x_H^P(\phi_H,\cdot)$ and $x_F^P(\phi_H,\cdot)$ are the quantities of intermediate inputs demanded by firm ϕ_H from domestic and international suppliers, respectively,[24] and σ_{HH}^P and σ_{HF}^P are the elasticities of substitution in the production of Home varieties between Home suppliers and Foreign suppliers, respectively. Final varieties from Foreign are produced using a CES aggregator over exported varieties from Home, so that

$$c_F(\phi_F) = \left[\int m_F(\phi_F, \phi_H) \left(\phi_{FH}^P x_F^P(\phi_F, \phi_H)\right)^{\frac{\sigma_{FH}^P-1}{\sigma_{FH}^P}} dG_{\phi_H}(\phi_H)\right]^{\frac{\sigma_{FH}^P}{\sigma_{FH}^P-1}}, \tag{10}$$

where $x_F^P(\cdot,\cdot)$ is the quantity exported by Home to Foreign firms; $\left(\phi_{FH}^P\right)'$ is an exporter-augmenting

[24] The superscript G denotes the fact that $\{x_i^G(\cdot)\}_{i=\{H,F\}}$ are aggregates of groups of varieties. The superscript P denotes the fact that $\{x_i^P(\cdot,\cdot)\}_{i=\{H,F\}}$ are at the firm pair level.

productivity term in Foreign's production when using exports from Home's firms and $\sigma_{FH}^P > 1$ is the elasticity of substitution between exported varieties in the Foreign country. As is clear from (9), the extensive margin of production networks plays a role in the aggregators $\{x_i^G(\cdot)\}_{i=\{H,F\}}$ and $c_F(\cdot)$.

Each country's representative household supplies L_i units of labor inelastically, owns firms in its own country, and has CES preferences over varieties,

$$Q_i = \left[\int (\phi_i^C c_i(\phi_i))^{\frac{\sigma_i^G-1}{\sigma_i^G}} dG_{\phi_i}(\phi_i)\right]^{\frac{\sigma_i^G}{\sigma_i^G-1}}, \quad i = \{H, F\}, \tag{11}$$

where σ_i^G is the elasticity of substitution between varieties in country i, $\sigma_i^G > 1$ is the elasticity of substitution between varieties in i, and ϕ_i^C is a quality shifter.[25]

There are two potential sources of non-linearity in the model. First, intermediate inputs are aggregated across suppliers (in $\{x_i^G(\cdot)\}_{i=\{H,F\}}$ and $c_F(\cdot)$). The previous research that estimates production functions has used similar functional forms in technology and demand but the authors usually assume the particular case of linear aggregation of intermediate inputs (Ackerberg et al., 2015; Gandhi et al., 2013).[26] That is a special case of my model, since it can be replicated with $\sigma_{FH}^P \to \infty$, $\sigma_{HH}^P \to \infty$ and $\sigma_{HF}^P \to \infty$. Second, elasticities of substitution are potentially heterogeneous in different margins. I depart from some part of the literature that imposes equal elasticities of substitution both within and across demand and production ($\sigma_H^G = \sigma_F^G = \sigma_{FH}^P = \sigma_{HH}^P = \sigma_{HF}^P = \sigma^X = \sigma^L$).[27] These two features introduce non-linearities in the model that are potentially consequential for the propagation of micro shocks, as argued in Baqaee and Farhi (2018a). These non-linearities are examined in the quantitative analysis in Section 5.

The market structure of the economy is one of monopolistic competition in the markets for final goods and inputs. As usual, this assumption implies that firms charge constant markups over marginal costs. Note that since the elasticity of substitution in demand is different from the one in production, standard estimates of markups such as revenue-to-cost ratios will vary across firms. In the model, markups at the firm level are a weighted combination of the markup from final and intermediates' demand, with different weights across firms. This heterogeneity arises because firms have different shares of revenues coming from final and intermediates' markets.[28,29]

[25] There is no standard way in the literature of how to name ϕ_i^C. In general, ϕ_i^C is anything that shifts households' demand conditional on price. In a CES setup, it is isomorphic to have households demand more of a good conditional on price because of its higher quality in some objective sense or because they have a higher subjective taste for it. Throughout the paper, I adopt the convention of calling the demand shifter ϕ_i^C *quality*, with the understanding that it could also be called *taste*.

[26] Some exceptions are Grieco et al. (2016) and De Loecker et al. (2016). Other exceptions are in the international trade literature (Antràs et al., 2017).

[27] For example, Lim (2017) and Gopinath and Neiman (2014).

[28] A similar implication is obtained in Kikkawa et al. (2017).

[29] This source of heterogeneity of markups between firms, due to their heterogeneous exposure to constant markups of different markets can be further studied with the transaction datasets used in this paper and the one from Belgium used in Tintelnot et al. (2018) and Kikkawa et al. (2017).

Given data constraints and the application to a small and open economy such as Chile, the analysis takes the aggregate from Foreign as given. That is, Foreign's aggregate demand shifter D_F is not solved endogenously.[30] In order to pin down D_F, I use the observed aggregate trade surplus at Home, which is defined as

$$\begin{aligned} TS_H &= \int\int m_F(\phi_F,\phi_H)p_F(\phi_F,\phi_H)x_F^P(\phi_F,\phi_H)dG_{\Phi_H}(\phi_H)dG_{\phi_F}(\phi_F) \\ &\quad - \int\int m_F(\phi_H,\phi_F)p_F(\phi_H,\phi_F)x_F^P(\phi_H,\phi_H)dG_{\phi_F}(\phi_F)dG_{\Phi_H}(\Phi_H), \end{aligned} \tag{12}$$

where $p_F(\phi_F,\phi_H)$ ($p_F(\phi_H,\phi_F)$) is the export (import) price of firm ϕ_H to (from) Foreign firm ϕ_F. Home household's aggregate expenditure of domestic final goods is

$$E_H = I_H - TS_H, \tag{13}$$

where I_H is Home's aggregate income.

The market clearing conditions for labor and each firm's output are given by

$$L_H - L_H^F - L_F^F \geq \int l(\phi_H)dG_{\phi_H}(\phi_H), \tag{14}$$

$$\begin{aligned} y(\phi_H) &\geq c_H(\phi_H) + \int m_F(\phi_F,\phi_H)x_F^P(\phi_F,\phi_H)dG_{\phi_F}(\phi_F) \\ &\quad + \int m_H(\phi'_H,\phi_H)x_H^P(\phi'_H,\phi_H)dG_{\phi_H}(\phi'_H), \quad \forall\phi_H \end{aligned} \tag{15}$$

where L_H^F and L_F^F are the aggregate mass of labor used to pay the fixed cost of relationships in intermediate input markets in Home and Foreign, respectively. Since in this section production networks' links are exogenous, $\{L_i^F\}_{i=\{H,F\}}$ are given. Each firm's output $y(\cdot)$ is sold to either domestic or foreign households, or to other domestic firms.

An equilibrium in the open economy with exogenous networks is defined in:

Definition 1. *Given the matching functions $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$ and the implied mass of labor used to form production links, $\{L_i^F\}_{i=\{H,F\}}$, an equilibrium in the* open economy with exogenous networks *is a set of allocation functions $\{y(\cdot), c(\cdot), l(\cdot), x(\cdot), x_H^G(\cdot), x_F^G(\cdot), x_H^P(\cdot,\cdot), x_F^P(\cdot,\cdot), c_H(\cdot), c_F(\cdot)\}$, prices $\{p_H^C(\cdot), p_F^C(\cdot), p_H(\cdot,\cdot), p_F(\cdot,\cdot)\}$, an ideal aggregate price index P_H, an aggregate trade surplus TS_H and a foreign demand shifter D_F such that firms maximize profits, the representative household maximizes utility subject to its budget constraint, and the trade surplus condition in (12) and market clearing conditions (14)-(15) are satisfied.*

3.1.2 Endogenous and Dynamic Production Network in an Open Economy

The previous section presented the model given the matching functions $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$. This section introduces the endogenous formation of these links. The formation and evolution of these relationships, and their aggregate implications, are the main object of analysis in this paper.

[30] This only comes from a data constraint. With all the necessary data, one could estimate and solve the model for Foreign similar to what I do for Home

Time is discrete and the domestic representative household has preferences defined by

$$U_t = \sum_{k=t}^{\infty} \beta^{k-t} Q_k,$$

where Q_k is the utility specified in (11) and β is the discount factor. Since the household's value function U_t is linear in each period's utility, its decision in each period is characterized by the one from the static model. The discount factor β only affects how firms discount the future, since they are owned by the household.

Two frictions characterize the formation of firm-to-firm relationships. First, there is a fixed cost that is paid for each relationship, at each time. Second, firms are exogenously not allowed to adjust every link over time. The technology specified in (7)-(9) implies that additional suppliers always reduce the marginal cost of the buyer, due to the love-for-variety structure of the CES technology.[31] On the other hand, since the technology is constant returns to scale, additional buyers do not affect the seller's marginal cost. Instead they increase the seller's revenue linearly. Thus, in the absence of frictions in forming relationships, firms always have incentives to increase their connections both upstream and downstream as much as possible, delivering a complete production network. The role of these frictions is to generate a non-trivial, and realistic, endogenous production network.

At any given time t, each seller must spend $\{f_{it}\}_{i=\{H,F\}}$ units of labor for each buyer it wants to have active. The assumption that the seller pays the fixed cost yields two simplifications. First, the constant markup condition from the static model also holds in the dynamic version. If the buyer had to pay a fraction of the fixed cost and found a relationship undesirable due to the CES markup charged by the seller, the seller could offer a lower price to incentivize the formation of the relationship. This assumption implies that firms are always willing to form upstream relationships, making buyers passive in the formation of relationships. Nevertheless, this argument only requires one side of the relationship to pay the fixed cost. Does it matter whether the buyer or the seller pays the fixed cost? If the buyers were to pay the fixed costs, making the sellers passive, they would have to solve an involved sourcing problem of finding the combination of suppliers that minimize their marginal cost. This sourcing problem is challenging because the aggregation of suppliers in the marginal cost is non-linear and thus when evaluating a mass of new suppliers at a given moment in time, the decisions made on other relationships might be altered (Antràs et al., 2017).[32] This problem does not appear when the seller pays the fixed cost because sellers aggregate buyers linearly in the revenue function, making the buyers perfect substitutes. This allows me to evaluate

[31] An alternative technology where instead of a love-for-variety technology, firms face convex costs of producing intermediate inputs from goods bought from suppliers would deliver the same marginal cost structure.

[32] In Antràs et al. (2017), the sourcing decision is more complex than here. In their model, varieties are discrete, which makes the sourcing of inputs a combinatorial problem that is challenging to solve for a large set of firms because one has to search over all the different combinations of input sets to find the one that minimizes costs. In this paper, the continuum of firms simplifies this problem. One does not have to evaluate all the combinations of different sets of varieties, since each one is mass zero. Nevertheless, the continuum does not eliminate the combinatorial problem entirely because at each moment in time, a positive mass of input types potentially changes. This, in turn, could affect the decision of other input types. Thus, in some sense the combinatorial problem goes from varieties to types, which is a higher level of aggregation.

each link separately.[33]

The fixed cost paid by the seller is given by $f_{it} = \delta_i \epsilon_{it}$, for $i = \{H, F\}$. The first term, δ_i, is time-invariant and captures the average level of relationship fixed costs.[34] The second term ϵ_{it} is a random variable that is independent and identically distributed across relationships within (i, t), with cumulative distribution G_{ϵ_i} and unit mean. The stochastic nature of ϵ_{it} generates the creation and destruction of production links over time, even in the steady state. Allowing for serial dependence would create another source of persistence, at the cost of increasing the computational difficulty in solving the model. Persistence in ϵ_{it} would require keeping track of an additional state that varies across firm pairs and time. Despite this simplifying assumption, the model generates non-trivial persistence in relationship formation, since firms face a non-pecuniary adjustment friction.

Firms are subject to an exogenous probability, $1 - \nu_i$, of whether they can reevaluate each relationship from i, at each moment in time.[35] This probability is independent over firm pairs and time.[36] This friction is a reduced-form approach to capture a variety of frictions that firms might face when creating or destroying relationships. For example, for creating relationships, firms might add new customers with probability less than one because it might be costly to find the buyer that meets the requirements of the sellers' output, or alternatively, it could be costly to adapt the output to the buyers' requirements. Also, the contract negotiation between firms might take time and be subject to the uncertainty of whether an agreement will be reached. On the destruction side, firms might face frictions in deactivating relationships because of legal barriers to terminating contracts or because the negotiation of ending the contract might be subject to uncertainty as well. Although firms face frictions in adding and dropping partners, they can adjust prices for all pairs at every moment in time, making the intensive margin completely flexible.

The two frictions of forming relationships imply that the evolution of link formation is governed by *chance* and *choice*. In order to reevaluate a link, firms need to have the chance to do so by receiving the shock $1 - \nu_i$. Although this is necessary for reevaluating a link, it is not sufficient to make the decision of what to do with the link. Conditional on receiving that shock, the firm has to choose whether to create or destroy that particular link. This part of the decision also has randomness due to the random component of relationships' fixed costs. Thus, given the chance to reevaluate a link, the probability of accepting that link is governed by $a_{Ht}(\phi_H, \phi'_H)$, the *acceptance function*. $a_{Ht}(\phi_H, \phi'_H)$ ($a_{Ft}(\phi_H, \phi_F)$) is the probability that firm ϕ'_H (ϕ_F) sells to ϕ_H, conditional on a realization that allows it.[37] The acceptance functions, $\{a_{it}(\cdot, \cdot)\}_{i=\{H,F\}}$, capture all the dynamic and strategic behavior when evaluating the formation and destruction of domestic and international

[33] This assumption is opposite from the one made in some of the previous literature (Antràs et al., 2017), where the buyer is active and pays the fixed cost entirely when forming relationships.

[34] Although it is assumed that δ_i is constant across firms and relationships among domestic and international margins, it can be accommodated to be heterogeneous across firms or relationships without increasing the computational complexity of solving the model.

[35] That is, the possibility of activating an inactive relationship or deactivating an active relationship.

[36] The assumption that ν_i is constant across firms within i is made for parsimony and can be easily extended to allow for heterogeneity across firms.

[37] Thus, it has a structure similar to that of the matching function $\{m_{it}(\cdot, \cdot)\}_{i=\{H,F\}}$.

relationships between firms.

Given all these assumption about the formation of links between firms, the matching function of domestic links evolves according to

$$\begin{aligned} m_{Ht}(\phi_H, \phi'_H) &= m_{Ht-1}(\phi_H, \phi'_H) \\ &\quad +[1 - m_{Ht-1}(\phi_H, \phi'_H)](1-\nu_H)a_{Ht}(\phi_H, \phi'_H) \\ &\quad -m_{Ht-1}(\phi_H, \phi'_H)(1-\nu_H)[1 - a_{Ht}(\phi_H, \phi'_H)], \quad \forall(\phi_H, \phi'_H), \end{aligned} \tag{16}$$

$$= \nu_H m_{Ht-1}(\phi_H, \phi'_H) + (1-\nu_H)a_{Ht}(\phi_H, \phi'_H), \quad \forall(\phi_H, \phi'_H). \tag{17}$$

The first term of the right-hand side of (16) represents the mass of relationships that were active in the previous period. The second term represents the mass of relationships that were inactive, the seller received the shock that allowed her to evaluate them and were ultimately accepted. The third term is the mass of relationships that were active, the seller received the shock that allowed her to evaluate them and were ultimately rejected. A similar logic applies to the evolution of the matching function of international links,

$$m_{Ft}(\phi_i, \phi_{i'}) = \nu_F m_{Ft-1}(\phi_i, \phi_{i'}) + (1-\nu_F)a_{Ft}(\phi_i, \phi_{i'}), \tag{18}$$

where $(i, i') = \{(H, F), (F, H)\}$.

Given the previous assumptions on the formation of production links, the Bellman equations for the creation and destruction of domestic production relationships are the following:

$$\begin{aligned} V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) &= \pi^P_{Ht}(\phi_H, \phi'_H) - \delta_H \epsilon_{Ht} \\ &\quad +\beta(1-\nu_H)\mathbb{E}_t V^R_{Ht+1}(\phi_H, \phi'_H|\epsilon_{Ht+1}) + \beta\nu_H \mathbb{E}_t V^A_{Ht+1}(\phi_H, \phi'_H|\epsilon_{Ht+1}), \end{aligned} \tag{19}$$

$$V^I_{Ht}(\phi_H, \phi'_H) = \beta(1-\nu_H)\mathbb{E}_t V^R_{Ht+1}(\phi_H, \phi'_H|\epsilon_{Ht+1}) + \beta\nu_H \mathbb{E}_t V^I_{Ht+1}(\phi_H, \phi'_H), \tag{20}$$

$$V^R_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) = \max\{V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}), V^I_{Ht}(\phi_H, \phi'_H)\}, \tag{21}$$

where $\pi^P_{Ht}(\phi_H, \phi'_H)$ is the gross profit that seller ϕ'_H obtains from the relationship with buyer ϕ_H, $V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht})$ is the value function of the (ϕ_H, ϕ'_H) relationship being active in t conditional on the fixed cost shock ϵ_{Ht}, $V^I_{Ht}(\phi_H, \phi'_H)$ is the value function when the relationship is inactive in t, and $V^R_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht})$ the value function of the seller ϕ'_H when she reevaluates the status of her relationship with buyer ϕ_H, conditional on ϵ_{Ht}. The structure of frictions in the formation of links between firms is the same for the formation of domestic and international links. Therefore, a similar value function for international relationships underlying firms' import and export activities can be found in Appendix B.2.

The following proposition characterizes the decisions to activate and terminate domestic links.[38]

Proposition 1. *The activation and termination of domestic production links are characterized as follows:*

[38] Similar activation and termination decisions hold for links of domestic firms with international firms. These can be found in Appendix B.2.

1. *If relationships are fully flexible in the extensive margin ($\nu_H = 0$) or firms are fully myopic ($\beta = 0$), then decisions are characterized by*

$$\begin{aligned} V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H) &= \pi^P_{Ht}(\phi_H, \phi'_H) - \delta_H \epsilon_{Ht}, \\ a_{Ht}(\phi_H, \phi'_H; \nu_H = 0 \mid \beta = 0) &= G_{\epsilon_H}\left[\frac{\pi^P_{Ht}(\phi_H, \phi'_H)}{\delta_H}\right]. \end{aligned} \tag{22}$$

2. *In the steady state where the functions π^P_{Ht}, V^R_{Ht}, V^A_{Ht} and V^I_{Ht} are constant, decisions are characterized by*

$$\begin{aligned} V^A_H(\phi_H, \phi'_H|\epsilon_H) - V^I_H(\phi_H, \phi'_H) &= \frac{\pi^P_H(\phi_H, \phi'_H) - \beta\nu_H\delta_H}{1 - \beta\nu_H} - \delta_H\epsilon_{Ht}, \\ a_H(\phi_H, \phi'_H) &= G_{\epsilon_H}\left[\frac{\pi^P_H(\phi_H, \phi'_H)/\delta_H - \beta\nu_H}{1 - \beta\nu_H}\right]. \end{aligned} \tag{23}$$

3. *Outside the steady state, decisions are characterized by*

$$\begin{aligned} V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H) &= \pi^P_{Ht}(\phi_H, \phi'_H) - \delta_H\epsilon_{Ht} \\ &\quad + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left[\pi^P_{Ht+k}(\phi_H, \phi'_H) - \delta_H\right], \end{aligned} \tag{24}$$

$$a_{Ht}(\phi_H, \phi'_H) = G_{\epsilon_H}\left[\frac{\pi^P_{Ht}(\phi_H, \phi'_H)}{\delta_H} + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left(\frac{\pi^P_{Ht+k}(\phi_H, \phi'_H)}{\delta_H} - 1\right)\right]. \tag{25}$$

Proposition 1 characterizes the activation and termination decisions in three scenarios. First, if there were no dynamics or the future did not matter, a firm's problem would be reduced to the static problem of evaluating whether the gross profits of activating a link exceed the fixed cost in each period. Once there are adjustment frictions ($\nu_H > 0$) and firms care about the future ($\beta > 0$), the problem of evaluating production links becomes forward looking. If a firm chooses not to change the activation or termination of a link today, it might need to wait several periods before having the chance to evaluate that decision again. Second, at the steady state, firms are willing to retain a link that is temporarily unprofitable, so long as it is profitable on average over time. For example, a firm would tolerate a link that has $\pi^P_H(\phi_H, \phi'_H) < \delta_H$ as long as $\pi^P_H(\phi_H, \phi'_H) > \beta\nu_H\delta_H$. Similarly, firms are willing to drop links that are temporarily profitable, if they are expected to be unprofitable on average over time. Firms tolerate links with steady-state profits as low as $\beta\nu_H\delta_H < \delta_H$.[39]

Finally, decisions outside the steady state take into account the expected future path of bilateral profits. This decision is the complex margin when solving the dynamics of firm-to-firm relationships:

[39] Note that Equation (23) shows how the acceptance function, and thus the matching function, can accommodate zeros in firm-to-firm links.

for each link, it is necessary to iterate on the full path of future bilateral profits along the transition path until the matching function converges to the new steady state. This is one of the main challenges when including dynamics in production networks. The exogeneity of the probability of whether a firm can reevaluate a link simplifies this problem. It implies that the discount factor in the decision to activate and terminate links is effectively $\beta\nu_H$, which, in turn, implies that discounting is a linear function of future profits. Despite this simplification, discounting the future profits of each link is still challenging. Appendix C describes the algorithm that solves this problem.

To finish characterizing the endogenous and dynamic production network, it is necessary to solve for the aggregate labor used in the fixed cost of active relationships, $\{L_{it}^F\}_{i=\{H,F\}}$, which was taken as given in the version of the model with exogenous networks in Section 3.1.1. Given that production networks are endogenous now, L_{Ht}^F is endogenously defined by:

$$\begin{aligned} L_{Ht}^F &= \int\int \nu_H m_{Ht-1}(\phi_H, \phi'_H)\delta_H \mathbb{E}_t\left(\epsilon_{Ht}\right) dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H) \\ &+ \int\int (1-\nu_H) a_{Ht}(\phi_H, \phi'_H)\delta_H \mathbb{E}_t\left(\epsilon_{Ht} | \epsilon_{Ht} < \bar{\epsilon}_{Ht}(\phi_H, \phi'_H)\right) dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H). \end{aligned}$$

The first integral represents the fixed costs paid by relationships that did not have the chance to be reevaluated, weighted by the mass of those relationships evaluated at the average fixed cost (since there is no selection on who has to pay for this group), which is $\delta_H = \mathbb{E}_t(f_{Ht})$. The second integral represents the fixed cost paid by relationships that have been reevaluated and were retained. As in a standard selection problem, since firms select into relationships, conditional on having the opportunity to do so, the value of the fixed cost for the relationships that are accepted depends on ϵ_{Ht} not being too large, thus the term $\mathbb{E}_t\left(\epsilon_{Ht} | \epsilon_{Ht} < \bar{\epsilon}_{Ht}(\phi_H, \phi'_H)\right)$. Proposition 1 implies that

$$\mathbb{E}_t\left(\epsilon_{Ht} | \epsilon_{Ht} < \bar{\epsilon}_{Ht}(\phi_H, \phi'_H)\right) = \int_0^{\bar{\epsilon}_{Ht}(\phi_H, \phi'_H)} \epsilon_{Ht} dG_{\epsilon_H}(\epsilon_H), \tag{26}$$

$$\bar{\epsilon}_{Ht}(\phi_H, \phi'_H) = \max\left\{\frac{\pi_{Ht}^P(\phi_H, \phi'_H)}{\delta_H} + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left(\frac{\pi_{Ht+k}^P(\phi_H, \phi'_H)}{\delta_H} - 1\right), 0\right\}. \tag{27}$$

Similar logic and formulas hold for thresholds in relationships between Home and Foreign firms and their respective aggregate fixed cost. These formulas can be found in Appendix B.2.

Having endogenized $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$ and $\{L_{it}^F\}_{i=\{H,F\}}$, an equilibrium in the open economy with endogenous and dynamic networks can be defined as follows:

Definition 2. *Given initial matching functions* $\{m_{i,-1}\}_{i=\{H,F\}}$ *an equilibrium in an* open economy with endogenous and dynamic networks *is a list of sequences of functions* $\{m_{it}(\cdot,\cdot), a_{it}(\cdot,\cdot), \pi_{it}^P(\cdot,\cdot)\}_{i=\{H,F\},t\in\mathbb{R}^+}$*, a sequence of aggregate price index* $\{P_{Ht}\}_{t\in\mathbb{R}^+}$*, a sequence of an aggregate trade surplus* $\{TS_{Ht}\}_{t\in\mathbb{R}^+}$ *and a sequence of aggregate foreign demand shifters* $\{D_{Ft}\}_{t\in\mathbb{R}^+}$ *such that firms maximize profits, the representative household maximizes utility subject to its budget constraint, and the trade surplus condition from* (12) *and market clearing conditions from* (14)-(15) *hold. Given* $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$ *and* $\{L_{it}^F\}_{i=\{H,F\}}$*, the allocation and prices at time t are defined as in Definition 1.*

The definition of the steady state follows directly:

Definition 3. *An equilibrium in the* steady state of the open economy with endogenous and

dynamic networks *is a list of functions* $\{m_i(\cdot,\cdot), a_i(\cdot,\cdot), \pi_i^P(\cdot,\cdot)\}_{i=\{H,F\}}$*, an aggregate price index* P_H*, an aggregate trade surplus* TS_H *and an aggregate foreign demand shifter* D_F *such that firms maximize profits, the representative household maximizes utility subject to its budget constraint, and the trade surplus condition from* (12) *and market clearing conditions from* (14)-(15) *hold. Given* $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$ *and* $\{L_{it}^F\}_{i=\{H,F\}}$*, the allocation and prices in the steady state are defined as in Definition 2.*

The model features several potential inefficiencies that can be important for the propagation of shocks as argued in Baqaee and Farhi (2018b). First, markets are monopolistically competitive, which introduces static distortions through markups. Second, the equilibrium does not feature an efficient bilateral bargaining protocol in production links. This is due to the assumption that the buyer is passive and, thus, is not allowed to make transfers to the seller in situations where the relationship is mutually beneficial but the seller is unwilling to pay the full fixed cost. Third, the model features network externalities, as shown in Lim (2017). These externalities reflect the fact that firms do not internalize the effect of creating and destroying their links on changes in the aggregate connectivity of the economy. Finally, the economy features inefficiencies that are standard in an open economy, as argued in Costinot et al. (2016) (e.g., the inefficiency from the planner having incentives to manipulate the terms of trade). The joint and separate implications of these inefficiencies for the propagation of shocks is left for future research.

4 Structural Estimation

This section describes the structural estimation of the model presented in Section 3.1.2, that is, the open economy with an endogenous and dynamic networks model. This section comprises three parts. The first describes the parametric assumptions. The second describes the estimation procedure. The third reports the estimation results.

4.1 Parametric Assumptions

In order to estimate the model, three parametric assumptions are needed: the distribution of Home and Foreign firms' primitives ϕ_i, the distribution of the stochastic component of relationships' fixed cost, and the distribution of international trade costs and shocks. I describe each assumption in turn.

As described in Section 3.1.1, firms' primitives in Home and Foreign, $\phi_H = (\phi_H^C, \phi_{FH}^P, \phi_H^L)$ and $\phi_F = (\phi_F^C, \phi_{HF}^P)$, are distributed according to an exogenous probability function denoted by G_{ϕ_H} and G_{ϕ_F}, respectively. Given that the distribution of firm size typically has a log-normal shape, I assume that ϕ_H and ϕ_F each have a joint log-normal distribution.

I assume that the stochastic component of relationships' fixed cost, $\{\epsilon_{it}\}_{i=\{H,F\}}$, is distributed as a Weibull (also called the Type III extreme value distribution), and is i.i.d. across firm pairs and time. There are three reasons for assuming this distribution. First, the Weibull distribution is a common distribution used for failure and hazard analysis. In the context of my model, it can be interpreted as the minimum of the cost draws for a given relationship. Second, it is related to

a standard distribution used in international trade, the Frechet distribution (Eaton and Kortum, 2002). The Frechet distribution is the inverse Weibull distribution. Thus, while Frechet distributions are usually used to represent the distribution of efficiency, the Weibull distribution is used here to represent the distribution of relationship costs. The third reason is for tractability. Given that, to solve the problem of link creation and destruction, one needs to compute the conditional mean of ϵ_{it}, e.g. see Equations (26) and (27), assuming a distribution that has a closed form for those expressions avoids integrating, which makes the computation of the dynamic problem simpler.[40] The Weibull distribution has two relevant parameters, a scale parameter and a shape parameter. Since δ_i is the unconditional mean of f_{it}, the scale parameter is chosen so that the unconditional mean of each $\{\epsilon_{it}\}_{i=\{H,F\}}$ is equal to one. The shape parameter is denoted by s_i. Finally, I assume, for simplicity, that the stochastic components of relationship fixed costs are independent across domestic and international relationships, i.e., $(\epsilon_{Ht}, \epsilon_{Ft})$ are independent from one another.

Finally, I assume that the international iceberg trade cost, τ_F, is distributed as a Pareto and the shocks to ϕ_F are distributed as a log-normal. These assumptions are made by backing out these primitives directly from the data using the structure of the model of Section 3.1. The procedure to implement this is described in Appendix D.4. Intuitively, these primitives are captured through prices and flows conditional on prices. The mapping between the data and these primitives is intermediated by the elasticities of substitution. Thus, the simulation of these primitives is included in the SMM described below. A lack of data on firm-to-firm level transactions between domestic and foreign firms prevents me from estimating τ_F and ϕ_F at the firm level. Thus, I assume that domestic firms form links with representative firms from international markets. International markets are defined at the 6-digit product-country pair, which is the most disaggregated data that is available. This assumption is made for both export and import activities.

4.2 Estimation Procedure

I first describe the parameters that are set exogenously. I then describe the simulated method of moments (SMM) used to estimate the remaining parameters.

Five sets of parameters are set exogenously. First, note from (27) that what matters for the dynamic decision of the firm is $\beta\nu_H$ since that is the relevant discounting factor that firms use in forming links. Thus, β and ν_H cannot be identified separately. Given that I conduct the empirical analysis at an annual frequency, I assume that $\beta = 0.96$ and estimate ν_H in the SMM. Second, since the model is scale invariant, I normalize labor supply to one, $L = 1$. Third, I assume that the mean of ϕ_H is zero. This assumption is without loss of generality because the mean of ϕ_H^L scales with α^L, the mean of ϕ_{FH}^P scales with α^I and the mean of ϕ_H^C scales with the level of welfare. Four, I assume that ϕ_H is independent from ϕ_F. That is, $cov(\phi_H^C, \phi_F^C)$, $cov(\phi_H^C, \phi_{HF}^P)$, $cov(\phi_{FH}^P, \phi_F^C)$, $cov(\phi_{FH}^P, \phi_{HF}^P)$, $cov(\phi_H^L, \phi_F^C)$ and $cov(\phi_H^L, \phi_{HF}^P)$ are all zero. This assumption is made for identification purposes. Otherwise, changes to ϕ_F over time would not be exogenous

[40] I show in Appendix B.5 that the conditional mean of the Weibull distribution has a simple and closed-form solution.

for domestic firms. Finally, I assume zero correlation between the iceberg trade cost and firms' international primitives. That is, $cov(\phi_F^C, \tau_F) = 0$ and $cov(\phi_{HF}^P, \tau_F) = 0$.

The remaining parameters of the model are estimated using a SMM technique. This procedure involves five sets of parameters. The first set includes parameters governing firms' primitives. Given the aforementioned parameterization, these include the variances and covariances of Home firms' primitives ($v(\phi_H^C)$, $v(\phi_{FH}^P)$, $v(\phi_H^L)$, $cov(\phi_H^C, \phi_{FH}^P)$, $cov(\phi_H^C, \phi_H^L)$, $cov(\phi_{FH}^P, \phi_H^L)$). The second set includes parameters describing Foreign firms' primitives and the international trade cost, ($v(\phi_F^C)$, $v(\phi_{HF}^P)$, $v(\tau_F)$, $cov(\phi_F^C, \phi_{HF}^P)$). The third set includes the elasticities of substitution, both in production and in consumption, $(\sigma^L, \sigma^X, \sigma_{HH}^P, \sigma_{HF}^P, \sigma_{FH}^P, \sigma_H^G, \sigma_F^G)$. The fourth set includes the weights in the CES production function (α^L, α^I).[41] The fifth set includes the parameters that characterize the matching functions, namely, the probability for reevaluating links, the shape of the Weibull distribution and the mean of the fixed cost for maintaining a relationship: $(\nu_H, \nu_F^E, \nu_F^I, s_H, s_F^E, s_F^I, \delta_H, \delta_F^E, \delta_F^I)$, where the superscript E (I) denotes the parameter on the export (import) margin.[42] In total, I estimate 28 parameters, which are stacked into the vector Θ. The details of the implementation of the SMM procedure are provided in Appendix D.1. In particular, given the large dimensionality of Θ, Appendix D.1 shows how the estimation can be simplified by using closed-form solutions from the model that exploit dependencies between the parameters. This simplifies the estimation, since it reduces the dimensionality of the set of parameters over which the algorithm iterates.

Five sets of moments are targeted to estimate Θ. The first set involves moments from the distribution of firms' employment, final sales, total sales and exports. Table D.5 reports these moments. The second set describes the relationship between foreign expenditure and prices, as reported in Table D.8 of Appendix D.4. The third set includes moments from the regressions of the propagation of international trade shocks presented originally in Table 8 from Section 2 and included in Table D.7. The fourth set includes the aggregate labor share and aggregate imports to domestic intermediate input expenditure (Table D.6). The fifth set includes moments from the cross-section and evolution of firms' linkages. These are shown in Figure D.2 and D.4.[43]

Although I estimate all parameters jointly in the SMM procedure, I discuss the rough intuitive connection between each set of moments and the corresponding parameters. The first set of moments is related to firms' size distribution. Note that if the production network is completely empty, then the primitives will completely shape the characteristics underlying that first set. Thus, the network introduces potential departures from the parameterization of firms' primitives. These are reported in the estimation results in Section 4.3. The second set of moments is related to foreign

[41] Usually these weights can be calibrated directly using analytical solutions of the model if the CES structure is normalized. Without normalizing, the CES weights are functions of the elasticities. In the setup of this model, the usual CES normalization does not work well due to the endogenous extensive margin. Thus, the CES weights are estimated jointly with the other parameters of the CES production function.

[42] For flexibility in matching the data, I assume that δ_F and s_F are different in export and import activities.

[43] Some of these facts have been presented in the previous literature but not using data from official and administrative sources. Bernard et al. (2015) use a dataset collected by a private company in Japan and Lim (2017) uses a dataset of a subset of suppliers of only publicly listed firms in the US.

Table 5: SMM Estimates - Primitives Distribution

Parameter	Related Moments	Estimated Parameter
$v(\phi_H^C)$	Var. Firms' Domestic Final Sales	0.73
$v(\phi_{FH}^P)$	Var. Firms' Exports Productivity	0.92
$v(\phi_H^L)$	Var. Firms' Employment	0.83
$cov(\phi_H^C, \phi_F^C)$	Cov. Firms' Domestic and Foreign Final Sales	0.64
$cov(\phi_H^C, \phi_H^L)$	Cov. Firms' Domestic Final Sales and Employment	0.21
$cov(\phi_{FH}^P, \phi_H^L)$	Cov. Firms' Exports Productivity and Employment	0.18
$v(\phi_F^C)$	Var. Exports Quality Shifter	0.73
$v(\phi_{HF}^P)$	Var. Import Prices	0.65
$v(\tau_F)$	Var. Trade Costs	0.41
$cov(\phi_F^C, \phi_{HF}^P)$	Cov. Exports Quality Shifter and Import Prices	0.23

Notes: Estimated parameter values of firms' primitives of Home and Foreign, and the moment that is related to each parameter.

primitives as Appendix D.4 explicitly shows. The third set of moments is related to the elasticities of substitution in production and demand. I show in Appendix D.5 the theoretical relationship between the elasticity of substitutions and firms' revenue function. Although I do not run regressions using the same structure of the revenue function, I run the auxiliary regressions specified in (4) that connect firms' revenues directly with shocks underlying the right-hand-side variables of the revenue function. Thus, I follow the strategy of indirect inference to estimate the elasticities of substitution by using as a target the auxiliary model given by those regressions. Finally, the moments of the cross-section and dynamics of firms' production links are targeted to be matched by adjusting the parameters of firms' relationship costs. Intuitively, the level of the number of links and its relationship in the cross-section with firms' size are related to firms' relationship fixed costs. The probability of reevaluating links, ν_i, is used to target how frequently firms adjust production links and how persistent these links are. Appendix D.2 shows how to produce these moments in the model, and thus how they are produced in the data. Note that given the chosen moments, it is not necessary to solve for the transition path in the SMM. This simplifies significantly the procedure of estimating the model.

4.3 Estimation Results

The results of the estimation are reported in Tables 5 and 6. There are three important takeaways from these results. First, despite not being imposed, elasticities of substitution increase with the level of aggregation in which they operate, both in production and in demand. This is intuitive and also consistent with previous results estimating nested CES structures (Redding and Weinstein, 2018). Furthermore, the elasticities of substitution fall within the range of estimates in the literature. Second, the stickiness of links with international markets is higher for exports than for imports. Finally, given the mean relationship cost parameters $\{\delta_i\}_{i=\{H,F\}}$, the share of employment dedicated to pay relationships' fixed cost is around 10 percent of total employment.

Table 6: SMM Estimates - Elasticities & Matching

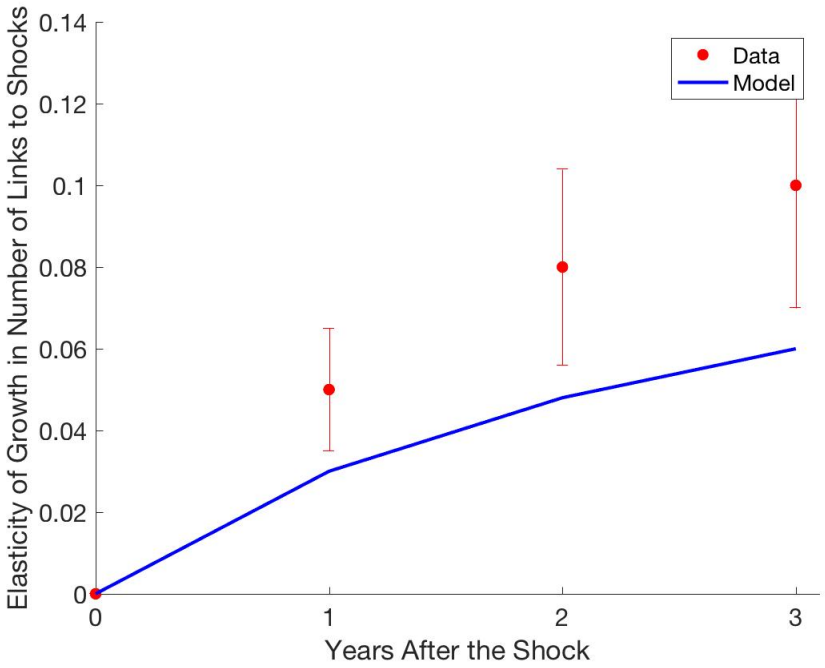
Parameter	Related Moments	Estimated Parameter
σ^L	Propagation Regressions	2.5
σ^X	Propagation Regressions	2.8
σ^P_{HH}	Propagation Regressions	3.4
σ^P_{HF}	International Trade Regression	3.1
σ^P_{FH}	International Trade Regression	4.2
σ^G_H	Propagation Regressions	3.8
σ^G_F	International Trade Regression	3.6
α^L	Aggregate Labor Share	0.7
α^I	Aggregate Import to Domestic Inputs	0.6
ν_H	Stickiness of Domestic Links	0.3
ν^E_F	Stickiness of Foreign Output Links (in exports)	0.5
ν^I_F	Stickiness of Foreign Input Links (in imports)	0.6
s_H	Size and Domestic Links' Stickiness	0.9
s^E_F	Size and Foreign Output Links' Stickiness (in exports)	1.2
s^I_F	Size and Foreign Input Links' Stickiness (in imports)	1.1
δ_H	Average Mass of Domestic Links	0.008
δ^E_F	Average Mass of Foreign Output Links (in exports)	0.3
δ^I_F	Average Mass of Foreign Input Links (in imports)	0.4

Notes: Estimated parameter values of elasticities, production networks' matching characteristics and CES weights, and the moments that intuitively are related to each parameter.

The quality of the estimation is tested by comparing the predictions of the model with non-targeted moments.[44] I use two sets of moments in these tests. First, related to Fact 6 on the sluggishness of firms' domestic links, Figure 4 reproduces the results from Figure 3, and adds the results of the same regression implemented in the model. The figure shows that the model is able to replicate that the number of domestic suppliers increases with import prices. It also shows that the reaction of the number of links is sluggish. Furthermore, the model captures about 60 percent of the sluggishness observed in the data. Thus, the model attributes less reaction to the international shocks than in the data, but the sluggishness seems to be similar. In fact, the 3-year to 1-year ratio of the elasticity of the number of suppliers to import prices is around 2 for both the model and the data. Through the lens of the model, firms expand their domestic links in response to import price shocks because those links become more expensive relative to domestic varieties. Although, due to data constraints, I cannot directly test for this since I do not observe the prices of domestic inputs, the mechanism in the model replicates the patterns in the data. The second non-targeted moment the elasticity of the number of links to small versus large international shocks. I test whether the model replicates Fact 7 documented in Section 2. The results are in Table 7. The model replicates that the reaction to large shocks is significantly larger than the one to small shocks.

[44] The fit of the estimation to targeted moments is discussed in Appendix D.3.

Figure 4: Sluggishness of Links' Reaction to Import Price Shocks: Model's Fit



Notes: This figure shows estimates of coefficients $\{\alpha_1^{I,h}\}_{h=1}^{3}$ of Equation (5) together with confidence intervals of 95 percent. The model's regression uses as shocks the direct variation of import price shocks.

Table 7: Small vs Large Shocks: Model's Fit

	Δ Log Number Buyers	
	Data	Model
Small Import Price Shock	0.159*	0.081
	[0.091]	
Large Import Price Shock	0.446**	0.321
	[0.191]	
Small Export Demand Shock	0.017*	0.004
	[0.010]	
Large Export Demand Shock	0.637***	0.464
	[0.114]	
N	13201	8320
R^2	0.708	0.541
Mean DV	0.44	0.51
SD DV	1.03	1.31

Notes: This table shows the results of the regressions specified in Equation (4), where the outcome is the log change in the number of buyers firms have. This regression is implemented in the data and in the model. The results of the data column are the same as the ones from Table 4 in Section 2. Large and small shocks are defined as shocks above and below the median of the size distribution of the shock, respectively. *** p<0.01, ** p<0.05, * p<0.1

5 Application: Propagation of the GR Trade Shocks in Chile

In this section, I use the estimated model to study the effects of the international trade shocks that Chile received during the Great Recession (GR) and evaluate how stickiness in production links influenced the propagation of those shocks. I describe the measurement of the shocks during the GR and then the results of the quantitative analysis.

5.1 International Trade Shocks During the Great Recession

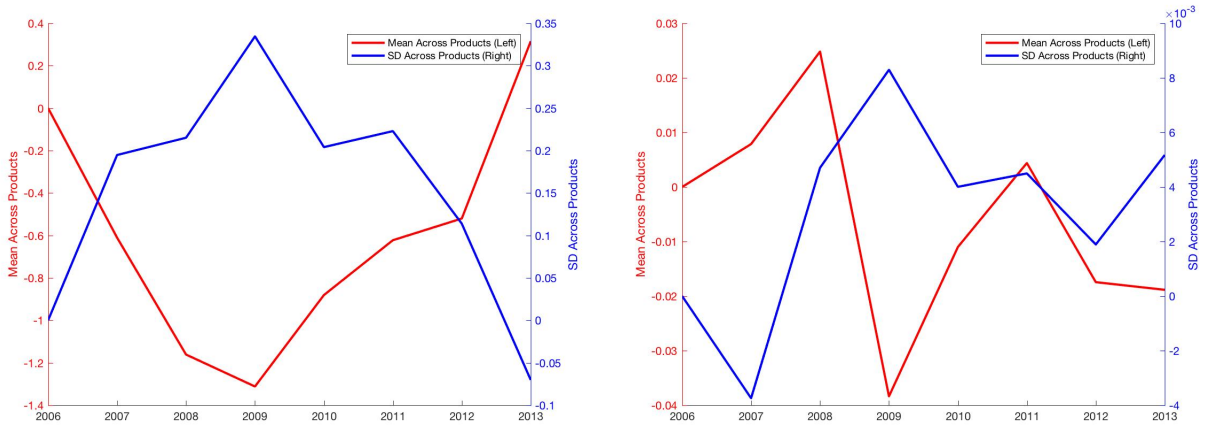
The sources of the GR were foreign and the effects were global. As a small and open economy, Chile was impacted by the GR through two margins: finance and international trade. I use the latter as exogenous shocks and evaluate how they propagated domestically through the production network. The firms that export and/or import from international markets were directly affected by the GR. Through the lens of the model, there are two potential margins by which the GR affected firms that were doing international trade. The first is through import prices, which can be directly measured in the data. Since the model features monopolistic competition and a constant returns to scale production function, prices are determined by the seller's marginal cost and not by market size. Thus, import price shocks are entirely exogenous from the perspective of Chilean firms and represent shocks to the productivity of foreign suppliers, i.e., shocks to ϕ^P_{HF}. The second is through export demand. Intuitively, changes in global output and international trade depressed import demand, affecting economies like Chile through a lower demand for its exported products. Export demand is measured using the structure of the model. It is represented by the foreign demand shifter ϕ^C_F, which shifts Foreign's demand for Home's varieties conditional on prices.[45] The theoretical details of how the export demand shock and import prices are derived from the model can be found in Appendix D.4. I show here evidence of the behavior of these two margins around the GR.

Figure 5 presents the export demand and import prices, around the GR, namely, between 2006 and 2013. The figures show the average and standard deviation of export demand and import prices across international markets that Chilean firms were exposed to during the 2000s. The figures show that the international trade shocks of the GR involved a decline in both average prices and demand, and an increase in their standard deviation. In terms of magnitudes, the average export demand declined by around 1.2 percent and its standard deviation increased by 30 log points, in 2009, the lowest point of the bust, relative to 2006, just before the GR began in the US.[46] In turn, the average import price declined not in 2006 but in 2008, by around 0.6 percent, while the standard deviation increased about 0.008 log points. The magnitudes in themselves are not straightforward

[45] Given that the model features monopolistic competition and thus constant markups, and a constant returns to scale production function, it ignores the international shocks to prices (flows) in exports (imports) since prices are determined by the origin's marginal cost and not by market size. One avenue for future research is to evaluate how much of the price variation could be captured by flow variation in a setup with variable markups.

[46] Although the GR started to affect the US in 2007, it affected Chile's international trade markets more meaningfully in and after 2009.

Figure 5: Export Demand and Import Prices - First and Second Moments Across Markets



(a) Export Demand

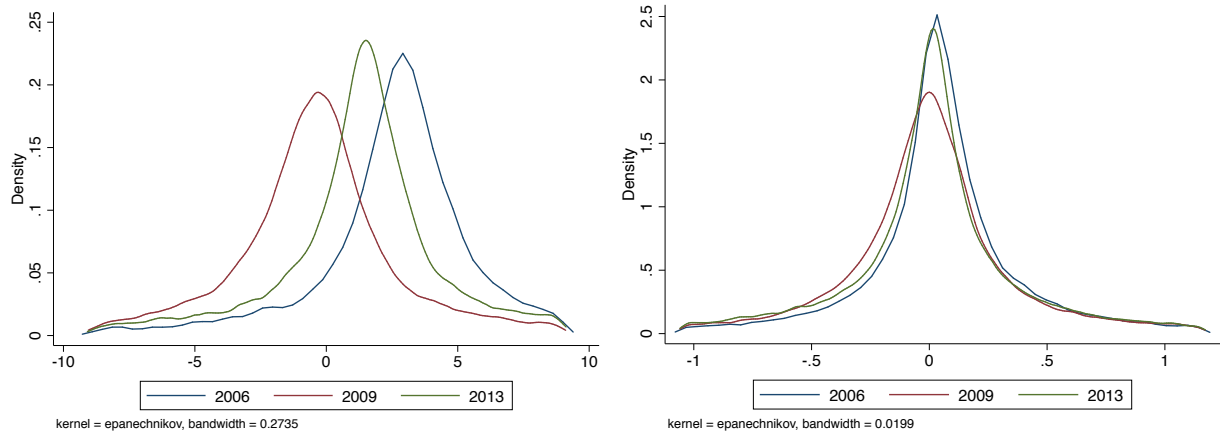
(b) Import Prices

Notes: These figures report the average and standard deviation of export demand shifters and import prices across international markets. Markets are defined as product-country combinations, and products are defined at the 6-digit HS code level of aggregation. Of around approximately 450 thousand international markets, only the ones that interact with Chile between 2006 and 2013 were considered (around 60 thousand). Details of how the export demand shifter and import prices were inferred from the data by using the model are explained in Appendix D.4.

to compare because they enter differently in the model, but the general pattern of decline in first moments and the increase in second moments hold on both margins. Another important conclusion from these figures is that in terms of export demand, the GR was over in 2012-2013. The variation of prices around this period is a bit more volatile so it is not clear exactly when these characteristics stabilized. The shape of the shocks is easier to understand when looking at the full distribution before and during the GR. Figure 6 presents the distribution of changes in the variables presented in Figure 5, for changes in 2005-2006, 2008-2009 and 2012-2013. This figure shows how at the lowest point of the bust, in 2009, the GR exhibited a decline in the mean and an increase in the dispersion of both export demand and import prices. In 2013, a significant part of the GR trade shocks were over, especially the one related to the second moment shock.

I implement the shock with a log-normal joint distribution of Hicks-neutral productivity shocks and demand shifters at the international market level. The moments used to measure these shocks are the ones from Figure 5. Given these shocks, the exercise proceeds as follows. I start from the estimated steady state which is simulated with 13,000 types of firms in Home and 40,000 types in Foreign. The challenges of simulating a model with a large set of firms is addressed in Appendix C, which explains the steps to simulate the model. An important step of the algorithm is the computation of the matrix of firm-to-firm relationship. In general, computing these matrices can be burdensome for an economy with a large number of firms. To avoid this challenge, I leverage the sparsity of firm-to-firm linkages. In the data, although firms have thousands of potential suppliers and customers, the average firm buys from around 30 suppliers and sells to 40 customers.

Figure 6: Great Recession International Trade Shocks - Full Distribution



(a) Export Demand Shock

(b) Import Price Shock

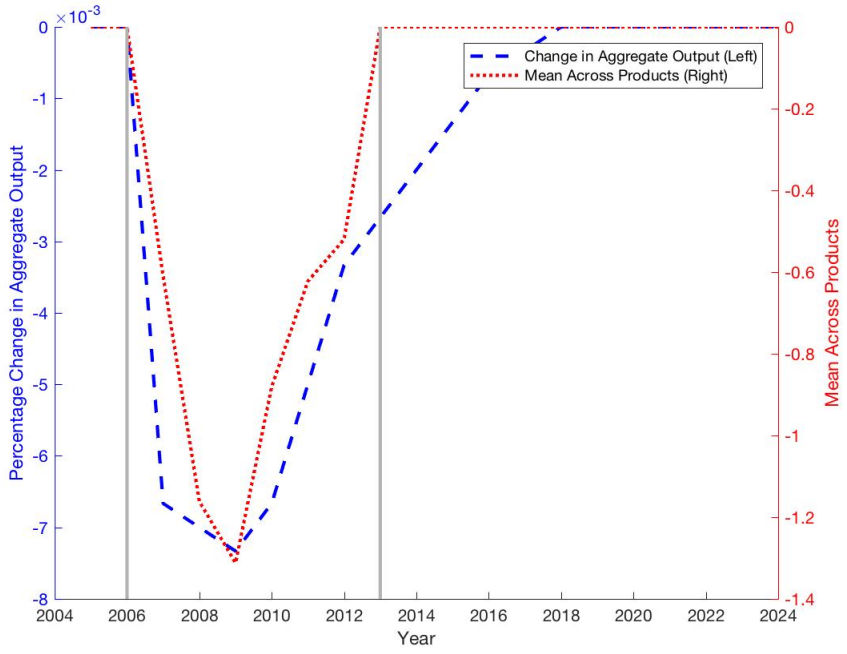
Notes: These figures show the distribution of first differences of export demand and import prices for 2005-2006, 2008-2009 and 2012-2013. These distributions show the variation in these two variables across international markets. Markets are defined as product-country combinations, and products are defined at the 6-digit HS code level of aggregation. Of approximately 450 thousand international markets, only the ones that interacted with Chile between 2006 and 2013 were considered (around 60 thousand). Details of how the export demand shifter and import prices were inferred from the data by using the model are explained in Appendix D.4.

This implies that instead of producing matrices that have information of all potential firm-to-firm relationships, I compute matrices that only keep track of the location and values of active links. These sparse matrices are standard tools in computational methods but has not been used in firm-to-firm analysis. Given the initial steady state, I assume that the GR was unanticipated by firms and was transitory, lasting for the period 2006-2012, as Figure 5 suggests. After 2012, the primitives return to their original values. Given these shocks, I evaluate how the model reacts to them and how much does the stickiness of production links affect the propagation of those shocks.

5.2 Results of the Propagation of the Great Recession Shocks

The simulated effect on Chilean firms of the international trade shocks during the GR appear in Figure 7. Aggregate output in the economy, which is the same as static aggregate welfare, declined by around 3 percent relative to the counterfactual where these shocks did not happen. One can see from the figure that aggregate output remains depressed around 5 years after the shock disappears, which highlights the sluggishness in production links. To address the role of the stickiness of production links, I compute the same outcomes but for an economy where firms can adjust their links flexibly, that is, an economy where $\nu_i = 0$, for all i. Such an economy would have seen a decline in aggregate output of 2 percent, one third less predicted by the model with the

Figure 7: Impulse Response Function of Present Discounted Value of Aggregate Consumption



Notes: This figure presents on the left axis percentage changes in output that come as a reaction to the international trade shocks. The right axis presents changes in the average of the export demand shifter ϕ_F^C, which corresponds to the same average presented in Figure 5. Grey lines denote the beginning and end of the GR.

estimated stickiness in production links. Without adjustment frictions in intermediate input links, Chile would have experienced a decline in output due to the international trade shocks during the GR by 30 percent less.

The GR affected aggregate Chilean output through several margins. First, firms engaging in international trade, either through importing or exporting, were directly affected by the trade shocks of the GR. This margin ignores all the indirect effects through the production networks. In the model, the part of the function $h(\cdot,\cdot)$ from Equation (51) with $d = 0$ represents this margin analytically. As a counterfactual, it is measured by evaluating changes in the quantities of these firms due to the shocks, holding prices fixed. Second, firms that did not engage in international trade before the GR might have been indirectly affected through the production network, given the production links that existed before the GR, with firms directly affected. That is, if a firm exports to a foreign market that received a negative demand shock, a domestic supplier of that firm that does not export might be negatively affected because the demand for its output might decline, and a domestic supplier of the firm's domestic supplier might also be affected despite the fact that it is not exporting and not affected directly by the shock. All these indirect effects are represented through the entire function $h(\cdot,\cdot)$, which measures paths between firms in the production network of all lengths. This margin is simulated by, starting from the initial steady state, setting $\nu_i = 1$ for

all i, that is, maximum persistence in production links. This second margin is usually decomposed into Hulten (1978)'s and Baqaee and Farhi (2018a)'s approximations, which captures the first-order and second-order terms, respectively, and higher-order terms. The third margin represents firms' adjustment of their production network to the shock, in the absence of adjustment friction. This margin is simulated with $\nu_i = 0$ for all i, that is, imposing no sluggishness in production links. Finally, production networks react sluggishly due to the stickiness in the evolution of production links. This margin is simulated by the full estimated version of the model. Note that, as the recent literature has pointed out (Baqaee and Farhi, 2018a,b), these last two components can be quantitatively relevant in this economy due to the non-linearities in production and demand as well as the potential inefficiencies the model has, such as the ones in the formation of these links.

Table 8 shows the results of the different propagation forces in the model. The direct effect accounts for about 9 percent of the total effect. The scenario with a fixed and exogenous production network generates an aggregate effect that is 18 percent larger than the case with the estimated stickiness of production links. Intuitively, in the case with an exogenous production network, firms have fewer margins of adjustment as a reaction to the shocks and thus the aggregate economy performs worse. The case with endogenous but totally flexible production networks has a lower effect than the case with an exogenous network, of around 50 percent. This is intuitive, since now firms can adjust and protect themselves from the international shock. The full model with the estimated stickiness of production links lies in between these extreme cases. The effect with an endogenous and flexible production network is around 30 percent smaller than the full estimated model, but the full model predicts around 20 percent lower effects than the case with a totally exogenous production network. These departures are quite significant and highlight the relevance of taking into account the endogeneity and frictions of firms' capacity to adjust their production links, when evaluating how shocks propagate.

A natural benchmark in the literature of the propagation of shocks is Hulten (1978)'s theorem. The theorem says that, in a closed economy, if allocations are efficient and there no non-linearities (which is accomplished by having Cobb-Douglas functional forms in both demand and production), then the size of the agent being affected by the shock is a sufficient statistic to measure the welfare effects of the shock. In other words, all the micro details that underlie how firms react are unnecessary to know. Hulten (1978)'s theorem is typically used with Hicks-neutral productivity shocks and thus the appropriate measure of size is firm's revenue to gross domestic product (GDP). In the context of the shocks in the application to the GR, I measure the effect of import productivity shocks as the share of expenditure in imports relative to GDP, and the effect of demand shocks to exports as the share of export revenues relative to GDP.[47] Computing these terms directly in the estimated initial steady state gives a total effect of both shocks of around 1.6 percent decline in GDP during the GR, which amounts to around 53 percent of the total effect estimated with the full model. This shows that both non-linearities and inefficiencies in the formation of these links can

[47] These measures ignore potential changes to terms of trade that these shocks might create as argued in Tintelnot et al. (2018). Considering those would require more assumptions or more data to compute them.

Table 8: Different Forces for the Propagation of Micro Shocks (%)

	Direct	Indirect		
		Exogenous: $\nu_H = 1$	Flexible: $\nu_H = 0$	Sticky: $\hat{\nu}_H = 0.3$
Full Model	9	118	66	100
Second Moment	16	106	60	100
Equal ES	13	113	81	100

Notes: This table presents the effects of the international trade shocks for different counterfactuals. *Full Model* refers to the counterfactuals using the fully estimated version of the model. *Second Moment* refers to counterfactuals where only second moment shocks are implemented, and not first moment shocks. Finally, *Equal ES* refers to counterfactuals of the estimated model with elasticities in demand and production equal to the one in final domestic demand ($\sigma = 3.8$), which is a value that is close to standard estimates in the literature. Each column of the table represents a percentage of the total output losses of the counterfactual by using different channels of propagation.

play an important departure from Hulten (1978)'s approximation.[48] The decomposition of these two mechanisms is left for future research.

When firms establish connections with other firms in intermediate input markets, they gain revenues by selling their output and reduce their marginal cost by supplying inputs. But when these connections are difficult to adjust, negative shocks to firms' linkages reduce firms' output because transitorily the firm is tied to a relationship she might prefer to destroy. Without these difficulties in adjusting production links, the trade shocks of the GR would have had a lower negative impact on the Chilean economy, by around 30 percent. This highlights that taking into account the sluggish reaction of production links is an important margin when evaluating the propagation of shocks.

I implement three exercises to further understand this result. First, I evaluate the separate role of international demand and supply shocks during the GR. Exporters were exposed to the former, whereas importers faced the latter. The import supply shock, which was a reduction in import prices, worked as a buffer for the negative demand shock that exporters faced. I evaluate the role of this buffer. I rerun the main counterfactual but leave ϕ^P_{HF} at the level it had in the initial steady state. I find that the total effect is increased to 3.6 percent output losses. This shows that the main effects of the GR were due to the demand channel through exporters and that the positive impact that the reduction in import prices could have had, was not so meaningful.

Second, I study the impact of the heterogeneity of the shock across firms. The only source of propagation in this model occurs with heterogeneous shocks. Aggregate Hicks-neutral shocks in this economy are not propagated in the sense that all firms are affected proportionally in the same magnitude. The application to the trade shocks of the GR had a component of an aggregate macro shock and also a micro shock, since firms perceived heterogenous magnitudes of the GR trade shocks. This is represented in the increase of the dispersion of both import prices and export demand. How much of the output losses are due to the macro component as opposed to the micro

[48] Tintelnot et al. (2018) also find that a back of the envelope calculation motivated by Hulten (1978)'s theorem gives widely different answers to how shocks propagate but in the context of endogenous but static production networks.

component?[49] I implement this by running a counterfactual in which only the second moment shocks are at work. I find that the total output losses from this case is around 2.1 percent, which implies that the second moment shocks account for around 2/3 of the total effect of the trade shocks during the GR. As Table 8 shows, without the adjustment costs and thus with an endogenous but totally flexible networks, the effects would have been about 40 lower. This exercise shows that the adjustment frictions play a more important role for the propagation of the micro shocks than the macro shock. In other words, it highlights that these frictions are important in the context of idiosyncratic shocks more than aggregate shocks.

Finally, I evaluate the role that different elasticities of substitution in demand and production have for the propagation of the trade shocks of the GR. Recall that the estimated version of the model features elasticities of substitution that are different in the different nests of the CES in demand and production. Since non-linearities can play an important role, it is relevant to understand whether it makes a difference the fact that these elasticities are allowed to be different. One of the reasons for this is that having different elasticities increases the complexity of solving the model and also limits the analytical solutions of the model. I implement the main exercise of the paper by setting all the elasticities equal to one another and equal to the estimated elasticity of substitution in Home's final demand ($\sigma = 3.8$), which is close to standard values in the literature. I find that, in this case, the propagation due to the adjustment friction is lower by about 15 percent, relative to the case with endogenous and flexible production networks, which amounts to around almost half of the propagation given by the case with different elasticities. This shows that allowing for different elasticities is important to have an accurate quantification of the role of adjustment frictions in the propagation of shocks.

6 Conclusions

Production networks are central for understanding how shocks propagate in an economy. Incorporating how costly it is to adjust the links of production networks is fundamental to evaluate how shocks propagate in general, and in particular how the international shocks experienced by Chile during the Great Recession propagated domestically. To address the aggregate and quantitative relevance of this feature, I provided micro reduced-form facts about firm-to-firm links' reaction to international shocks and replicated these features in a general equilibrium model with production network dynamics. The estimated model shows that without adjustment frictions in firm-to-firm relationships, the GR international trade shocks would have had a negative effect on output of about 30 percent less. This highlights that any analysis of the propagation of external shocks that aims to be quantitatively accurate should incorporate dynamic effects arising from frictions in the adjustment of production links.

[49] In practice, even the macro component of the international trade shock is heterogeneous across firms because only a subset of firms is affected by it: the set that selects into international trade. Nevertheless, in the context of this model, it is still important to understand how relevant in international trade macro shocks (represented by first moment shocks) are relative to micro shocks (represented by second moment shocks).

By providing a quantitative evaluation of how endogenous and dynamic production networks propagate shocks, my research opens up a range of further questions. In particular, three questions are natural given this framework. First, how much of these adjustment frictions are due to technological constraints as opposed to sourcing constraints? Since the model can accommodate industry level production networks, cross-industry differences in production functions can be included to isolate the role of adjustment frictions of sourcing from adjustment friction because of technology differences between firms across industry. Second, understanding in more detail the inefficiencies in the formation of firm-to-firm linkages and how that affects the propagation of shocks can be important for designing stabilization policies in intermediate input markets that help firms to adjust their linkages over the business cycle. Third, the increasing reliance on global value chains induces firms and countries to be more connected with one another and thus more dependent on their policy decisions. Adjustment frictions in the formation of international linkages underlying these global value chains can further increase the global propagation of shocks and, in particular, the global propagation of trade policies. In a context of rising protectionism of trade, this paper shows that these policy changes may be evaluated differently when taking into account the frictions in forming and adjusting international linkages between firms and countries.

References

Acemoglu, D., U. Akcigit, and W. Kerr (2016). Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual 30*(1), 273–335.

Acemoglu, D. and P. D. Azar (2017, December 2017). Endogenous production networks. *National Bureau of Economic Research Working Paper Series No. 24116*.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. *Econometrica 80*(5), 1977–2016.

Ackerberg, D. A., K. Caves, and G. Frazer (2015, 2018/09/03). Identification properties of recent production function estimators. *Econometrica 83*(6), 2411–2451.

Adão, R., M. Kolesár, and E. Morales (2018, August 2018). Shift-share designs: Theory and inference. *National Bureau of Economic Research Working Paper Series No. 24944*.

Alfaro, L., P. Antras, D. Chor, and P. Conconi (2015). Internalizing global value chains: A firm-level analysis. *Working Paper*.

Antràs, P. and A. de Gortari (2017, May 2017). On the geography of global value chains. *National Bureau of Economic Research Working Paper Series No. 23456*.

Antràs, P., T. C. Fort, and F. Tintelnot (2017, September). The margins of global sourcing: Theory and evidence from us firms. *American Economic Review 107*(9), 2514–64.

Atalay, E., A. Hortaçsu, J. Roberts, and C. Syverson (2011, 03). Network structure of production. *Proceedings of the National Academy of Sciences 108*(13), 5199–5202.

Auer, R., A. Levchenko, and P. Saure (2018). International inflation spillovers through input linkages. *Working Paper*.

Autor, D. H., D. Dorn, and G. H. Hanson (2013, October). The china syndrome: Local labor market effects of import competition in the united states. *American Economic Review 103*(6), 2121–68.

Bachmann, R., R. J. Caballero, and E. M. R. A. Engel (2013, October). Aggregate implications of lumpy investment: New evidence and a dsge model. *American Economic Journal: Macroeconomics 5*(4), 29–67.

Ballester, C., C.-A. Antoni, and Z. Yves (2006, 2018/07/29). Who's who in networks. wanted: The key player. *Econometrica 74*(5), 1403–1417.

Baqaee, D. R. and E. Farhi (2018a, January 2018). The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *National Bureau of Economic Research Working Paper Series No. 23145*.

Baqaee, D. R. and E. Farhi (2018b, March 2018). Productivity and misallocation in general equilibrium. *National Bureau of Economic Research Working Paper Series No. 24007*.

Barrot, J.-N. and J. Sauvagnat (2016). Input specificity and the propagation of idiosyncratic shocks in production networks*. *The Quarterly Journal of Economics*.

Bernard, A., T. Fort, V. Smeets, and F. Warzynski (2018). Heterogeneous globalization: Offshoring and reorganization. *Working Paper*.

Bernard, A. B., E. A. Bøler, and S. Dhingra (2018, April 2018). Firm-to-firm connections in colombian imports. *National Bureau of Economic Research Working Paper Series No. 24557*.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2009, May). The margins of us trade. *American Economic Review 99*(2), 487–93.

Bernard, A. B. and A. Moxnes (2018, April 2018). Networks and trade. *National Bureau of Economic Research Working Paper Series No. 24556*.

Bernard, A. B., A. Moxnes, and Y. U. Saito (2015, April 2015). Production networks, geography and firm performance. *National Bureau of Economic Research Working Paper Series No. 21082*.

Bernard, A. B., A. Moxnes, and K. H. Ulltveit-Moe (2014, May 2014). Two-sided heterogeneity and trade. *National Bureau of Economic Research Working Paper Series No. 20136*.

Boehm, C., A. Flaaen, and N. Pandalai-Nayar (2017). Input linkages and the transmission of shocks: Firm-level evidence from the 2011 tohoku earthquake. *Working Paper*.

Bramoullé, Y., R. Kranton, and M. D'Amours (2014, March). Strategic interaction and networks. *American Economic Review 104*(3), 898–930.

Caballero, R. J., E. M. R. A. Engel, and J. Haltiwanger (1997). Aggregate employment dynamics: Building from microeconomic evidence. *87*(1), 115–137.

Caliendo, L. and F. Parro (2015, 01). Estimates of the trade and welfare effects of nafta. *The Review of Economic Studies 82*(1), 1–44.

Caliendo, L., F. Parro, E. Rossi-Hansberg, and P.-D. Sarte (2014, May 2014). The impact of regional and sectoral productivity changes on the u.s. economy. *National Bureau of Economic Research Working Paper Series No. 20168*.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). Peer effects and social networks in education. *The Review of Economic Studies 76*(4), 1239.

Carvalho, V., M. Nirei, Y. U. Saito, and A. Tahbaz-Salehi (2016). Supply chain disruptions: Evidence from great east japan earthquake. *Working Paper*.

Carvalho, V. and N. Voigtlander (2015). Input diffusion and the evolution of production networks. *Working Paper*.

Chaney, T. (2014, November). The network structure of international trade. *American Economic Review 104*(11), 3600–3634.

Costinot, A., A. Rodríguez-Clare, and I. Werning (2016, February 2016). Micro to macro: Optimal trade policy with firm heterogeneity. *National Bureau of Economic Research Working Paper Series No. 21989*.

Costinot, A., J. Vogel, and S. Wang (2013, 01). An elementary theory of global supply chains. *The Review of Economic Studies 80*(1), 109–144.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2016, 2018/08/02). Prices, markups, and trade reform. *Econometrica 84*(2), 445–510.

di Giovanni, J., A. Levchenko, and I. Mejean (2018a). Foreign shocks as granular fluctuations. *Working Paper*.

di Giovanni, J., A. Levchenko, and I. Mejean (2018b, January). The micro origins of international business-cycle comovement. *American Economic Review 108*(1), 82–108.

Eaton, J., D. Jinkins, J. Tybout, and D. Xu (2016). Two-sided search in international markets. *Working Paper*.

Eaton, J. and S. Kortum (2002, 2018/09/18). Technology, geography, and trade. *Econometrica 70*(5), 1741–1779.

Eaton, J., S. Kortum, and F. Kramarz (2016). Firm-to-firm trade: Imports, exports and the labor market. *Working Paper*.

Economist-Intelligence-Unit (2009). Managing supply-chain risk for reward. Technical report, The Economist.

Fally, T. and R. Hillberry (2015, September 2015). A coasian model of international production chains. *National Bureau of Economic Research Working Paper Series No. 21520*.

Fieler, A. C. and A. Harrison (2018, April 2018). Escaping import competition and downstream tariffs. *National Bureau of Economic Research Working Paper Series No. 24527*.

Gandhi, A., S. Navarro, and D. Rivers (2013). On the identification of production functions: How heterogeneous is productivity? *Working Paper*.

Goldsmith-Pinkham, P., I. Sorkin, and H. Swift (2018, March 2018). Bartik instruments: What, when, why, and how. *National Bureau of Economic Research Working Paper Series No. 24408*.

Gopinath, G. and B. Neiman (2014, March). Trade adjustment and productivity in large crises. *American Economic Review 104*(3), 793–831.

Gourio, F. and L. Rudanko (2014, 07). Customer capital. *The Review of Economic Studies 81*(3), 1102–1136.

Grieco, P. L. E., S. Li, and H. Zhang (2016). Production function estimation with unobserved input price dispersion. *International Economic Review 57*(2), 665–690.

Heise, S. (2018). Firm-to-firm relationships and the pass-through of shocks: Theory and evidence. *Working Paper*.

Horvath, M. (1998). Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics 1*(4), 781–808.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *45*(3), 511–518.

Hummels, D., R. Jørgensen, J. Munch, and C. Xiang (2014, June). The wage effects of offshoring: Evidence from danish matched worker-firm data. *American Economic Review 104*(6), 1597–1629.

Huo, Z., A. Levchenko, and N. Pandalai-Nayar (2018). The global business cycle: Measurement and transmission. *Working Paper*.

Jackson, M. O. (2010). *Social and Economic Networks*. Princeton University Press.

Jordà, Ò. (2005, March). Estimation and inference of impulse responses by local projections. *American Economic Review 95*(1), 161–182.

Kikkawa, A. K., G. Magerman, and E. Dhyne (2017). Imperfect competition and the transmission of shocks: The network matters. *Working Paper*.

Lim, K. (2017). Firm-to-firm trade in sticky production networks. *Working Paper*.

Liu, E. (2018). Industrial policies in production networks. *Working Paper*.

Long, J. B. and C. I. Plosser (1983). Real business cycles. *91*(1), 39–69.

Lu, D., A. Mariscal, and L.-F. Mejia (2017). How firms accumulate inputs: Evidence from import switching. *Working Paper*.

Magerman, G., E. Dhyne, and S. Rubínová (2015). The belgian production network 2002-2012. *Working Paper Research 288, National Bank of Belgium*.

Monarch, R. (2016). "it's not you, it's me": Breakups in u.s.-china trade relationships. *International Finance Discussion Papers, N1165*.

Nakamura, E. and J. Steinsson (2013, 2018/11/09). Price rigidity: Microeconomic evidence and macroeconomic implications. *Annual Review of Economics 5*(1), 133–163.

Oberfield, E. (2018, 2018/09/13). A theory of input–output architecture. *Econometrica 86*(2), 559–589.

Redding, S. and D. Weinstein (2018). Accounting for trade patterns. *Working Paper*.

Song, J., D. L. Price, F. Guvenen, N. Bloom, and T. von Watcher (2018). Firming up inequality. *Working Paper N. 618, Princeton University, Industrial Relations Section*.

Taschereau-Dumouchel, M. (2018). Cascades and fluctuations in an economy with an endogenous production network. *Working Paper*.

Tintelnot, F., A. K. Kikkawa, M. Mogstad, and E. Dhyne (2018). Trade and domestic production networks. *Working Paper*.

Zou, Y. (2018). Endogenous production networks and gains from trade. *Working Paper*.

Appendix A Supporting Facts

A.1 More Production Network Facts

In this subsection I complement the analysis from the main text with more production networks facts. These facts support the main arguments of the paper even further. In particular, facts B.2. and B.3. highlight the relevance of working with firm-level datasets in production networks, as opposed to industry-level.

Fact B.1 *Production links' locations and industries account for similar shares in the extensive margin of intermediate input markets.* To show this, I implement a slight variation of the decomposition from Equation (1). Consider firm i's total intermediate input expenditures or revenues, x_i, decomposed as:

$$x_{it} \quad = \quad n_{it}^{L} n_{it}^{I} c_{it} \bar{x}_{it}, \tag{28}$$

where n_{it}^{L} is the number of locations in which firm i has production links in (either suppliers or buyers) in year t, n_{it}^{I} the number of 6-digit industries from which firm i has production links, $c_{it} = \frac{n_{it}^{P}}{n_{it}^{L} n_{it}^{I}}$ is the effective number of partners that firms have, n_{it}^{P}, relative to the potential number of location-industries combination. Consider c_{it} as a measure of how concentrated the number of links are across location-industries combinations. Note that the first three components of Equation (28) represent the role of the extensive margin. The fourth component, $\bar{x}_{it} = x_{it}/n_{it}^{P}$, is firms' average expenditure or revenue across partner firms, that is, the intensive margin of intermediate input markets. Since in log terms, these components add up to x_{it}, I use ordinary least squares (OLS) to decompose the variation of $\log(x_{it})$ into these four components. Table A.1 presents the results for 2011. It shows that for both expenditures and revenues from intermediate inputs, the extensive margin accounts for around 53 percent of the variation between firms. Relatively equal shares of this 53 percent come from the variation in the number of locations and industries firms source from and sell to.

Fact B.2 *The extensive margin accounts for a significant part of the variation in expenditures and revenues from intermediate inputs when implemented at the firm level. The opposite pattern holds when the level of aggregation increase up until 1 digit industries, where the majority of the variation comes from the intensive margin.* I implement the same decomposition from Table A.1 and Equation (28), but for different levels of aggregation. Table A.2 presents the evidence. The first row replicates the evidence from the upper panel of Table A.1. The rest of the rows implement the same decomposition for different levels of aggregation. One can see that as one goes from lower levels of aggregation to higher levels up until 1-digit industries, the intensive margin matters more and more.

Fact B.3 *Industries account for a small fraction of the variation of flows in firms' production networks, even within narrowly defined industry pairs.* Given that much of the production networks literature works at the industry level, I test how much of the variation of flows in the production

Table A.1: Cross-Sectional Decomposition of Firms' Expenditures and Revenues in Intermediate Input Markets (2011)

	Locations	Industries	Concentration	Average
Log. Input Expenditures	0.253***	0.301***	-0.025***	0.471***
	(0.003)	(0.002)	(0.003)	(0.003)
N	18425	18425	18425	18425
R^2	0.754	0.721	0.653	0.762
	Locations	Industries	Concentration	Average
Log. Input Revenues	0.229***	0.341***	-0.031***	0.461***
	(0.003)	(0.003)	(0.002)	(0.002)
N	18425	18425	18425	18425
R^2	0.732	0.763	0.684	0.721

Notes: OLS regressions of the log of Equation (28) implemented for 2011. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.2: Cross-Sectional Decomposition of Expenditures and Revenues in Intermediate Input Markets (2011)

	Locations	Industries	Concentration	Average
Firms	25.3	30.1	-0.025	47.1
Industries 6-D	20.6	22.6	0.9	56.0
Industries 5-D	19.4	20.3	2.7	57.6
Industries 4-D	17.5	13.9	9.2	59.5
Industries 3-D	16.2	9.3	14.4	60.2
Industries 2-D	15.6	6.1	15.2	63.1
Industries 1-D	5.7	0.3	27.6	66.4

Notes: OLS regressions of the log of Equation (28) implemented for 2011. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

network between firms is accounted for by industry variation. To evaluate this, I implement the following regression:

$$\log v_{ijt} \quad = \quad \gamma_{k(i)h(j)t} + \beta \log s_{it} + \epsilon_{ijt}, \tag{29}$$

where v_{ijt} is the intermediate input flow payment going from firm j to i at time t, s_{it} is sales of firm i at time t and $\gamma_{k(i)h(j)t}$ are industry pair-time fixed effects where firm i belongs to industry k and firm j to industry h. The goal of Equation (29) is to evaluate how much do $\gamma_{k(i)h(j)t}$ accounts for the variation of intermediate input flows v_{ijt} between firms. In other words, what percentage of the total variation of flows between firms occurs within the pair of industries the firms are involved in, for a particular year, relative to the variation between industry pairs. In Equation (29) I control for the sales of i because I want to take into account firm size when looking at the variation of the flows. Therefore, Equation (29) is similar to having at the left hand side the coefficient of

Table A.3: Decomposition of Input Flows Between Firms into Industry Variation

	Log Intermediate Input Flows Between Firms						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log Sales Buyer	0.283***	0.241***	0.261***	0.267***	0.266***	0.263***	0.262***
	[0.031]	[0.013]	[0.010]	[0.009]	[0.009]	[0.008]	[0.008]
1-Digit Industry Pair-Year FE		✓					
2-Digit Industry Pair-Year FE			✓				
3-Digit Industry Pair-Year FE				✓			
4-Digit Industry Pair-Year FE					✓		
5-Digit Industry Pair-Year FE						✓	
6-Digit Industry Pair-Year FE							✓
N (millions)	48.2	48.2	48.2	48.2	48.2	48.2	48.2
R^2	0.112	0.130	0.174	0.196	0.208	0.221	0.222

Notes: OLS regressions of Equation (29) implemented for 2003-2011. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

an input-output table at the firm level and evaluate how much the industry input-output table accounts for the variation in the firm input-output table.[50] Results are in Table A.3. It shows that industries cannot account for the variation of firm-to-firm intermediate input flows by more than 22 percent, shown in the R^2 from column 7. This occurs when using the most narrowly defined industry classification, at the 6-digit level, which corresponds to around 650 industries.[51] Furthermore, column 1 of Table A.3 shows that even without industry pair-year fixed effects, the sales of the buyer accounts for around 11 percent of the variation of v_{ijt}, which is about half of the 22 percent accounted for by industries. Thus, industries cannot account for more than around 11 percent of the variation of flows between firms in intermediate input markets. This highlight the immense variation unaccounted for by industry input-output tables.

Fact B.4 *Productivity Shocks Propagate Also Upstream, not Only Downstream.* Table 3 also shows that downstream import price shocks propagate also upstream. If a firm has a buyer that faces a positive shock on its import prices, then the firm will see its revenues increased. This is consistent with there being some level of substitution from import demand of the buyer to a higher domestic demand, which increases revenues of the upstream supplier. This level of substitution is not perfect though, because if it were, we would not see a direct negative effect of the own import shock on revenues since firms would substitute perfectly towards domestic suppliers. This is the first piece of evidence of difficulties in substituting foreign with domestic links. Furthermore, it is an indication of inefficiencies in the economy. Baqaee and Farhi (2018b) show that in an efficient environment the effect on output of a productivity shock to firm i only depends on its role as a supplier, whereas if the economy is inefficient it also depends on its role as a customer.

[50] One advantage of the specification in Equation (29), compared to a specification where the left hand side variable is the coefficients of an input-output table, is that the left hand side variable in (29) is continuous.

[51] The industry classification used in these regressions is the one used by the SII. It corresponds to a slightly modified version of *ISIC Rev. 4*.

Table A.4: Validity of Trade Shocks: Anticipating International Trade Shocks

	Import Shares	Export Shares
	(1)	(2)
Δ Log Price Shock	0.00897	
	(0.0145)	
Δ Log Flow Shock		-0.0207
		(0.0263)
Firm-Year FE	✓	✓
Overall R^2	0.448	0.482
Within R^2	0.000	0.000
N	878719	149104

Notes: OLS regressions of Equation (30) in column (1) and of Equation (31) in column (2). As in the empirical analysis where the shift-share shocks are used (Section 2), variables are censored at the 1 and 99 percent.

A.2 More Validations of the International Trade Shocks

This appendix performs a validation test of the assumption of exogeneity of the shocks. The idea is that the exogeneity would be violated if, at the time firms' make the decision of which international markets to connect to, they anticipate future changes in prices or demand of those markets, that is, the shifts of the shock are already in firms' information set when they choose the shares of markets they will be exposed to. If that is the case, the shares in the instrument would be correlated with the future shocks, making the effect of the shock no longer exogenous. I test this in the data by measuring that correlation. Specifically, I run the following specifications:

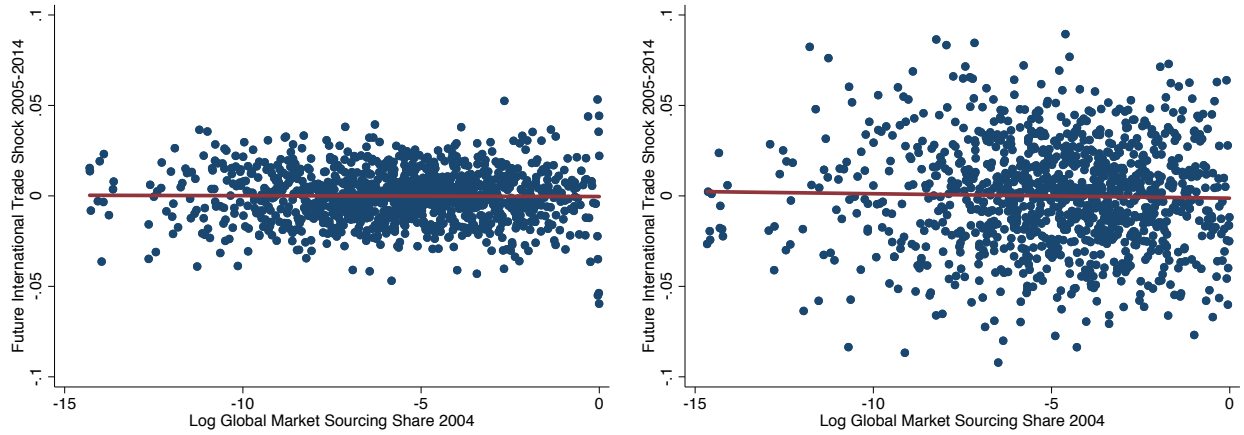
$$\log s^I_{ik,0} = \alpha^I_1 + \alpha^I_2 \Delta \log p^G_{k,t} + \gamma^I_{i,t} + \epsilon^I_{ik,t}, \quad k \in \Omega^I_{i,0}, \tag{30}$$

$$\log s^E_{ik,0} = \alpha^E_1 + \alpha^E_2 \Delta \log d^G_{k,t} + \gamma^E_{i,t} + \epsilon^E_{ik,t}, \quad k \in \Omega^E_{i,0}, \tag{31}$$

where the variables are taken from the specifications in equation (2) and (3),[52] except for $\gamma^I_{i,t}$ and $\gamma^E_{i,t}$ which are firm-year fixed effects, and $\epsilon^I_{ik,t}$ and $\epsilon^E_{ik,t}$ which are residuals at the firm-market-year level. The coefficient of interest is α^I_2 and α^E_2. They capture to what extent firms allocate more shares at time $t = 0$ to markets that have lower price growth or higher demand in the future. Table A.4 and Figure A.1 presents the results. Both pieces of evidence show that there is no significant correlation. This mitigates the concern that the shares in the shift-share design already have internalized future changes of the shift and supports the assumption of exogeneity of the shifts.

[52] As a reminder, $p^G_{k,t}$ is the average export price of international market k at time t excluding the price exported to Chile from that market, $d^G_{k,t}$ is the flow imported from market k at time t from the rest of the world excluding Chile, $s^I_{ik,0}$ ($s^E_{ik,0}$) is the share of firm i's total imports (exports) that comes (goes) from (to) international market k at time $t = 0$. Finally, $\Omega^I_{i,0}$ and $\Omega^E_{i,0}$ is the set of markets that firm i imports from and exports to at time 0, respectively.

Figure A.1: Correlation of Current Shares and Future International Trade Shocks



(a) Import Price Shock

(b) Export Demand Shock

Notes: OLS regressions of Equation (30) in Panel (a) and of Equation (31) in Panel (b). Firm-Year fixed effects have been extracted so that the variation presented is within firms, across markets, each year. The regression is implemented at the percentile-year level. That is, since the dataset underlying these regression is too large, it is collapsed in 100 bins within each year, ordered according to the variable in the X axis.

Appendix B Model

This section describes the open economy model in more detail. The purpose is to explicitly present all the details of the derivations and to gain intuition on how the model works.

B.1 Exogenous Production Network in an Open Economy

The solution to Home firms' cost minimization problem is

$$c_H(\phi_H) = \left[\left(\alpha^L\right)^{\sigma^L} \left(w/\phi_H^L\right)^{1-\sigma^L} + \left(1-\alpha^L\right)^{\sigma^L} p^{X,T}(\phi_H)^{1-\sigma^L} \right]^{\frac{1}{1-\sigma^L}}, \tag{32}$$

$$p^{X,T}(\phi_H) = \left[p_H^{X,G}(\phi_H)^{1-\sigma^X} + \left(\alpha^I\right)^{\sigma^X} p_F^{X,G}(\phi_H)^{1-\sigma^X} \right]^{\frac{1}{1-\sigma^X}}, \tag{33}$$

$$p_i^{X,G}(\phi_H) = \left[\int m_i(\phi_H, \phi_i') p_i^P(\phi_H, \phi_i')^{1-\sigma_{Hi}^P} dG_{\phi_i}(\phi_i) \right]^{\frac{1}{1-\sigma_{Hi}^P}}, \tag{34}$$

$$x^T(\phi_H) = \left(1-\alpha^L\right)^{\sigma^L} p^{X,T}(\phi_H)^{-\sigma^L} c(\phi_H)^{\sigma^L} y(\phi_H), \tag{35}$$

$$x_H^G(\phi_H) = p_H^{X,G}(\phi_H)^{-\sigma^X} p^{X,T}(\phi_H)^{\sigma^X} x^T(\phi_H), \tag{36}$$

$$x_F^G(\phi_H) = \left(\alpha^I\right)^{\sigma^X} p_F^{X,G}(\phi_H)^{-\sigma^X} p^{X,T}(\phi_H)^{\sigma^X} x^T(\phi_H), \tag{37}$$

$$x_i^P(\phi_H, \phi_i') = p_i^P(\phi_H, \phi_i')^{-\sigma_{Hi}^P} p_i^{X,G}(\phi_H)^{\sigma_{Hi}^P} x_i^G(\phi_H), \tag{38}$$

$$l(\phi_H) = \left(\alpha^L\right)^{\sigma^L} \left(\phi_H^L\right)^{\sigma^L - 1} w^{-\sigma^L} c_H(\phi_H)^{\sigma^L} y(\phi_H), \tag{39}$$

where $c_H(\phi_H)$ is Home firm ϕ_H's marginal cost, wages are the numeraire so that $w = 1$, $p^{X,T}(\phi_H)$ is the price index of ϕ_H's intermediate input expenditures across all domestic and international sources,[53] $p_i^{X,G}(\phi_H)$ is the price index across varieties supplied from country i, $p_i^P(\phi_H, \phi_i')$ is the bilateral price of varieties bought by ϕ_H to ϕ_i'.

The solution to Home's household problem and Foreign's demand are

$$c_i(\phi_i) = \left(\phi_i^C\right)^{\sigma_i^G - 1} p_i^C(\phi_i)^{-\sigma_i^G} D_i, \; i = \{H, F\}, \tag{40}$$

$$x_F^P(\phi_F, \phi_H) = \left(\phi_{FH}^P\right)^{\sigma_{FH}^P - 1} p_F^P(\phi_F, \phi_H)^{-\sigma_{FH}^P} p_F^C(\phi_F)^{\sigma_{FH}^P} c_F(\phi_F), \tag{41}$$

$$p_F^C(\phi_F) = \left[\int m_F(\phi_F, \phi_H) \left(p_F^P(\phi_F, \phi_H)/\phi_{FH}^P\right)^{1-\sigma_{FH}^P} dG_{\phi_H}(\phi_H)\right]^{\frac{1}{1-\sigma_{FH}^P}}, \tag{42}$$

$$P_i = \left[\int \left(p_i^C(\phi_i)/\phi_i^C\right)^{1-\sigma_i^G} dG_{\phi_i}(\phi_i)\right]^{\frac{1}{1-\sigma_i^G}}, \tag{43}$$

where $p_i^C(\phi_i)$ is the price that ϕ_i charges to i's household, D_i is the aggregate demand shifter from country i, defined as

$$D_i \equiv E_i P_i^{\sigma_i^G - 1}, \tag{44}$$

and $p_F^P(\phi_F, \phi_H)$ is the bilateral price when ϕ_H sells to ϕ_F.

Given the market structure of monopolistic competition in intermediate inputs and final demand, prices are

$$p_i^P(\phi_H, \phi_i') = \underbrace{\frac{\sigma_{Hi}^P}{1 - \sigma_{Hi}^P}}_{\mu_{Hi}^P} c_i(\phi_i), \tag{45}$$

$$p_F^P(\phi_F, \phi_H) = \underbrace{\frac{\sigma_{FH}^P}{1 - \sigma_{FH}^P}}_{\mu_{FH}^P} \tau_F c_H(\phi_H), \tag{46}$$

$$p_H^C(\phi_H) = \underbrace{\frac{\sigma_H^G}{1 - \sigma_H^G}}_{\mu_H^C} c_H(\phi_H), \tag{47}$$

where $c_F(\phi_F)$ is the marginal cost of the foreign firm, τ_F is the iceberg trade cost, and μ_{Hi}^P, μ_{FH}^P, μ_H^C are the markups when Home firms buy imports, sell to Foreign firms and sell to Home's household, respectively. Given the iceberg trade cost τ_F and Foreign's firm productivity, one has that $c_F(\phi_F) = \tau_F/\phi_{HF}^P$.

The firm-to-firm bilateral profits gross of the fixed costs and profits from sales to the household are

[53] Note that this price varies across firms since firms are connected to an heterogeneous set of firms.

$$\pi_i^P(\phi_i,\phi'_{i'}) = \frac{p_i^P(\phi_i,\phi'_{i'})x_i^P(\phi_i,\phi'_{i'})}{\sigma_{ii'}^P},\ i=\{H,F\},\ i'=\{H,F\}, \tag{48}$$

$$\pi_i^C(\phi_i) = \frac{p_i^C(\phi_i)c_i(\phi_i)}{\sigma_i^G},\ i=\{H,F\}, \tag{49}$$

where $\pi_i(\phi_i,\phi'_i)$ are the bilateral gross profits when ϕ'_i sells to ϕ_i. Net profits of ϕ_H are

$$\begin{aligned}\pi_H(\phi_H) = &\sum_{i=\{H,F\}}\pi_H^C(\phi_H)+\int m_H(\phi'_H,\phi_H)\pi_H^P(\phi'_H,\phi_H)dG_{\phi_H}(\phi'_H) \\ &-\sum_{i=\{H,F\}}\int a_{it}(\phi'_i,\phi_H)\delta_i\mathbb{E}_t\left(\epsilon_{it}|\epsilon_{it}<\bar{\epsilon}_{it}(\phi'_i,\phi_H)\right)dG_{\phi_i}(\phi'_i),\end{aligned} \tag{50}$$

where the third expression from the right hand side represents the fixed costs paid in forming domestic and international firm-to-firm relationships.

In order to formalize the ingredients of the network that are used throughout the paper, a couple of definitions are in order. Define the production network $\mathbf{n}$ with nodes $n(\phi_H,\phi'_H) = m_H(\phi_H,\phi'_H)g_{\phi_H}(\phi'_H)$. Define also the function

$$h(\phi_H,\phi'_H;\mathbf{n},\gamma)=\sum_{d=0}^{\infty}\gamma^d n^{[d]}(\phi_H,\phi'_H), \tag{51}$$

with scalar γ,[54] where $g^{[d]}(\phi_H,\phi'_H)\geq 0$ measures the mass of paths of length $d\geq 0$ in $\mathbf{n}$ from ϕ_H to ϕ'_H, where paths of length d are weighted by γ^d.[55] Thus, $h(\phi,\phi';\mathbf{n},\gamma)$ measures the total mass of paths between ϕ_H and ϕ'_H, of all possible lengths, in $\mathbf{n}$ weighted by γ.[56]

As in a standard firm model, firms' output is determined by their own sources of heterogeneity, ϕ_i in this case. Nevertheless, the production network implies that each firm's output is also determined by the heterogeneity of firms' suppliers and buyers. In turn, their suppliers and buyers are influenced potentially by their own links with other firms, so on and so forth. Thus, given the matching functions, $\{m_i(\cdot,\cdot)\}_{i=\{H,F\}}$, the model implies a potentially complex interaction between firms that makes it difficult to establish firms' output and efficiency, the key endogenous objects of firms, in terms of fundamentals. The following proposition addresses this challenge when all elasticities are identical.

Proposition 2. *Given a network* $\mathbf{n}$*, if* $\sigma_H^G=\sigma_F^G=\sigma_{FH}^P=\sigma_{HH}^P=\sigma_{HF}^P=\sigma^X=\sigma^L=\sigma$*, then firms' marginal cost, denoted by* $c_H(\cdot)$*, and output in Home are*

[54] An object similar to the elements $h(\phi_H,\phi'_H;\mathbf{n},\gamma)$ is often defined in the literature as the entries of the Leontief inverse matrix, e.g. Baqaee and Farhi (2018b). I refrain from that denomination in this section since the former is not defined in terms of expenditures and does not refer to a discrete set of nodes, as the latter does.

[55] In fact, γ is the factor that scales down the weight of longer paths. More details behind function $h(\cdot,\cdot;\mathbf{n},\gamma)$ are discussed in Appendix B.3.

[56] To simplify the notation, the reference to $\mathbf{n}$ is avoided when using $h(\cdot,\cdot;\mathbf{n},\gamma)$.

$$c_H(\phi_H) = \left[\int h(\phi_H,\phi'_H;\gamma^c)\left(\frac{1}{\bar{\phi}^P(\phi'_H)}\right)^{1-\sigma} dG_{\phi_H}(\phi'_H)\right]^{\frac{1}{1-\sigma}}, \tag{52}$$

$$\frac{y(\phi_H)}{D_H} = c_H(\phi_H)^{-\sigma}\mu^{-\sigma}\int h(\phi'_H,\phi_H;\gamma^y)\left(\bar{\phi}^C(\phi'_H)\right)^{\sigma-1} dG_{\phi_H}(\phi'_H), \tag{53}$$

where $\bar{\phi}^C(\phi_H)$ *is a firm type* ϕ_H *average demand shifter across different sources,*[57] $\bar{\phi}^P(\phi_H)$ *is a firm type* ϕ_H *average productivity shifter across different sources,*[58] $D_i \equiv Q_i P_i^\sigma$ *is the aggregate demand shifter of country* i*,* P_i *the aggregate price index of country* i *and* $\mu = \frac{\sigma}{\sigma-1}$ *is the constant markup. Finally,* $\gamma^c = \mu\left(\frac{1-\alpha^L}{\mu}\right)^\sigma$ *and* $\gamma^y = \left(\frac{1-\alpha^L}{\mu}\right)^\sigma$ *are the scalars of function* $h(\cdot,\cdot)$*.*

Proposition 2 characterizes how the production network's extensive margin influences firms' efficiency as measured by the marginal cost and firms' output.[59] I argue that this way of representing the marginal cost and output is informative in the context of networks theory. In the networks literature (e.g. Jackson (2010), Ballester et al. (2006)), for node ϕ_H (a firm type in this case), the measure of weighted Bonacich centralities of parameter γ in network $\mathbf{n}$ is defined in this context as $b_z(\phi_H;\mathbf{n},\gamma) = \int h(\phi_H,\phi'_H;\gamma)z(\phi'_H)dG_{\phi_H}(\phi'_H)$, where the function $z(\cdot)$ represents the characteristic that weights the connections between nodes. This measures the *total* mass of paths in $\mathbf{n}$, across all firms in the network, that start at ϕ_H, weighted by the characteristic $z(\cdot)$.[60] Thus marginal cost can be defined as a productivity-weighted Bonacich centrality and output as quality-to-productivity-weighted Bonacich centrality.[61] In short, they will be denoted as *productivity centrality* and *quality-to-productivity centrality*. These measures highlight that each firm's marginal cost and output are a combination of the primitive productivity and quality of all firms linked directly, indirectly and through any possible path in the production network.[62]

Given that Proposition 2 describes how production network affects each firm, Proposition 3 describes how this, in turn, influences aggregate welfare.

Proposition 3. *Given the network* $\mathbf{n}$ *and the implied* $\{L_i^F\}_{i=\{H,F\}}$*, if* $\sigma_H^G = \sigma_F^G = \sigma_{FH}^P = \sigma_{HH}^P = \sigma_{HF}^P = \sigma^X = \sigma^L = \sigma$ *and* $\mu = 1$*, then home's aggregate welfare is* $Q_H = A_H L_H$ *and aggregate*

[57] $\bar{\phi}^C(\phi_H) = \left[\left(\phi_H^C\right)^{\sigma-1} + \int m_F(\phi_F,\phi_H)\left(\phi_F^C\phi_{FH}^P\right)^{\sigma-1}\tau_F^{-\sigma}D_F/D_H dG_{\phi_F}(\phi_F)\right]^{\frac{1}{\sigma-1}}$. One can see in this expression, that demand shocks are, to a power function, isomorphic to inverse iceberg trade costs.

[58] $\frac{1}{\bar{\phi}^P(\phi_H)} = \left[\left(\alpha^L\right)^\sigma\left(\frac{1}{\phi_H^L}\right)^{1-\sigma} + (1-\alpha^L)^\sigma\mu^{1-\sigma}\int m_F(\phi_H,\phi_F)\left(\frac{\tau_F}{\phi_{HF}^P}\right)^{1-\sigma} dG_{\phi_F}(\phi_F)\right]^{\frac{1}{1-\sigma}}$

[59] The assumption $\sigma_H^G = \sigma_F^G = \sigma_{FH}^P = \sigma_{HH}^P = \sigma_{HF}^P = \sigma^X = \sigma^L = \sigma$ is necessary to allow for closed form solutions. Nevertheless, the intuition of Proposition 2 goes through with different elasticities of substitution. Actually, similar intuitions can be reached with first order approximations as in Baqaee and Farhi (2018a). In any case, the quantitative section will allow for arbitrary differences in these elasticities so the condition of this proposition is only for illustration and intuition purposes.

[60] It has been shown that this object is important for characterizing a set of network games (Bramoullé et al., 2014).

[61] Note that α^L and μ govern the rate of decay of the importance of more distant paths.

[62] This result resembles the ones in Acemoglu et al. (2012), with the difference that here production and final demand are generalized. I depart from their assumption of Cobb-Douglas functional forms on technology and demand into CES, and allow for heterogeneity not only in productivity but also in final demand. Finally, I also allow for international trade.

productivity is:

$$A_H \equiv \left(\frac{L_H - L_H^F - L_F^F}{L_H}\right) \underbrace{\frac{\left[\int\int h(\phi_H, \phi'_H; \gamma^y) \left(\phi_H^C \bar{\phi}^P(\phi'_H)\right)^{\sigma-1} dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H)\right]^{\frac{\sigma}{\sigma-1}}}{\int\int h(\phi_H, \phi'_H; \gamma^y) \left(\bar{\phi}^C(\phi_H)\phi_H^{L\prime}\right)^{\sigma-1} dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H)}}_{\mathscr{C}_H}. \quad (54)$$

In the absence of international trade, $\mathscr{C}_H$ is defined as:

$$\mathscr{C}_H^{closed} \equiv \left(\int\int h(\phi_H, \phi'_H; \gamma^y) \left(\phi_H^C \phi_H^{L\prime}\right)^{\sigma-1} dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H)\right)^{\frac{1}{\sigma-1}}. \quad (55)$$

Proposition 3 describes, under the assumption of equal elasticities, how the production network affects aggregate welfare and, in particular, aggregate productivity. Aggregate productivity can be decomposed into the share of labor used for variable costs of production, the first term of Equation (54), and the connectivity between firms, the second term of Equation (54), denoted by $\mathscr{C}_H$ and $\mathscr{C}_H^{closed}$ in the open and closed economy, respectively. It is in this object that all the production network is summarized. To gain intuition, I describe the main features of this aggregate connectivity in the closed economy. These features also apply for the open economy. In the closed economy, the aggregate connectivity consists of the aggregation of firms' production and demand primitives weighted by how well connected firms are with each other. Equation (55) has two important implications. The first is that it highlights that in a model with production networks, not only idiosyncratic productivity matters for aggregate productivity but also idiosyncratic quality since the former has more effect when it is better connected with the latter. $\mathscr{C}_H^{closed}$ shows that aggregate connectivity, and thus aggregate productivity, is higher when sellers of greater productivity $\phi_H^{L\prime}$ are closer connected to downstream buyers of greater quality ϕ_H^C. Intuitively, sellers of greater productivity will have lower marginal costs and prices which, given the curvature in demand, expands the production of buyers. This, in turn, will be more harnessed when the buyer has higher quality. The second, is that improving the matching between firms improves aggregate productivity, even if idiosyncratic primitives in productivity and quality are given. As before, the key measure to evaluate the connectivity between firms is the function $h(\cdot, \cdot; \gamma^y)$. Aggregate connectivity and productivity increases when the mass of paths that connect firms with each other is higher, that is, when $h(\phi_H, \phi'_H; \gamma^y)$ is higher for the pair (ϕ_H, ϕ'_H).[63] This is one of the main contributions of this paper, which is explored theoretically in Section 3.1.2 through endogeneizing $h(\cdot, \cdot; \gamma^y)$ and in the quantitative Section 5 by evaluating how $h(\cdot, \cdot; \gamma^y)$ evolves as a reaction to foreign shocks. A similar logic applies to the open economy, but where foreign demand and productivity primitives enter into the aggregate connectivity measure.

[63] As before, this is related to standard measures from the networks literature. Aggregate connectivity can be denoted as an aggregate *quality-to-productivity centrality*.

B.2 Endogenous and Dynamic Production Network in an Open Economy

The Bellman equations of the formation of international links are the following:

$$\begin{aligned}
V^A_{Ft}(\phi_i, \phi'_{i'}|\epsilon_{Ft}) &= \pi_{Ft}(\phi_i, \phi'_{i'}) - \delta_F \epsilon_{Ft} \\
&\quad + \beta(1-\nu_F)\mathbb{E}_t V^R_{Ft+1}(\phi_i, \phi'_{i'}|\epsilon_{Ft+1}) + \beta\nu_F \mathbb{E}_t V^A_{Ft+1}(\phi_i, \phi'_{i'}|\epsilon_{Ft+1}), \\
V^I_{Ft}(\phi_i, \phi'_{i'}) &= \beta(1-\nu_F)\mathbb{E}_t V^R_{Ft+1}(\phi_i, \phi'_{i'}|\epsilon_{Ft+1}) + \beta\nu_F \mathbb{E}_t V^I_{Ft+1}(\phi_i, \phi'_{i'}), \\
V^R_{Ft}(\phi_i, \phi'_{i'}|\epsilon_{Ft}) &= \max\{V^A_{Ft}(\phi_i, \phi'_{i'}|\epsilon_{Ft}), V^I_{Ft}(\phi_i, \phi'_{i'})\},
\end{aligned}$$

where $(i, i') = \{(H, F), (F, H)\}$, $\pi_{Ft}(\phi_i, \phi'_{i'})$ is the gross profit that seller $\phi'_{i'}$ obtains from the relationship with buyer ϕ_i, $V^A_{Ft}(\phi_i, \phi'_{i'}|\epsilon_{Ft})$ is the value function of the $(\phi_i, \phi'_{i'})$ relationship being active in t conditional on the fixed cost shock ϵ_{Ft}, $V^I_{Ft}(\phi_i, \phi'_{i'})$ is the value function when the relationships is inactive in t, and $V^R_{Ft}(\phi_i, \phi'_{i'}|\epsilon_{Ht})$ the value function of the seller $\phi'_{i'}$ when she reevaluates the status of her relationship with buyer ϕ_i, conditional on ϵ_{Ft}. Note that these Bellman equations govern the evolution of linkages between firms in import and export activities. This means that the structure of frictions in adjusting links is the same domestically and internationally.

The characterization of the value functions in the dynamic problem of link formation between Home and Foreign firms follows the same structure of the characterization of links between domestic firms in Proposition 1, and is presented in the following proposition.

Proposition 4. *The activation and termination of production links between Home and Foreign firms is characterized as follows:*

1. *If relationships are fully flexible in the extensive margin ($\nu_F = 0$) or firms are fully myopic ($\beta = 0$), then decisions are characterized by*

$$\begin{aligned}
V^A_{it}(\phi_i, \phi'_{i'}|\epsilon_{Ft}) - V^I_{it}(\phi_i, \phi'_{i'}) &= \pi_{it}(\phi_i, \phi'_{i'}) - \delta_F \epsilon_{Ft}, \\
a_{it}(\phi_i, \phi'_{i'}; \nu_F = 0 \mid \beta = 0) &= G_{\epsilon_F}\left[\frac{\pi_{it}(\phi_i, \phi'_{i'})}{\delta_F}\right].
\end{aligned}$$

2. *In the steady-state where the functions π_{it}, V^R_{it}, V^A_{it} and V^I_{it} are constant, decisions are characterized by*

$$\begin{aligned}
V^A_i(\phi_i, \phi'_{i'}|\epsilon_F) - V^I_i(\phi_i, \phi'_{i'}) &= \frac{\pi_i(\phi_i, \phi'_{i'}) - \beta\nu_F\delta_F}{1-\beta\nu_F} - \delta_F\epsilon_{Ft}, \\
a_i(\phi_i, \phi'_{i'}) &= G_{\epsilon_F}\left[\frac{\pi_i(\phi_i, \phi'_{i'})/\delta_F - \beta\nu_F}{1-\beta\nu_F}\right]. \qquad (56)
\end{aligned}$$

3. *Outside the steady-state, decisions are characterized by*

$$V_{it}^{A}(\phi_i,\phi'_{i'}|\epsilon_{Ft}) - V_{it}^{I}(\phi_i,\phi'_{i'}) = \pi_{it}(\phi_i,\phi'_{i'}) - \delta_F\epsilon_{Ft} + \sum_{k=1}^{\infty}(\beta\nu_F)^k\left[\pi_{it+k}(\phi_i,\phi'_{i'}) - \delta_F\right],$$

$$a_{it}(\phi_i,\phi'_{i'}) = G_{\epsilon_F}\left[\frac{\pi_{it}(\phi_i,\phi'_{i'})}{\delta_F} + \sum_{k=1}^{\infty}(\beta\nu_F)^k\left(\frac{\pi_{it+k}(\phi_i,\phi'_{i'})}{\delta_F} - 1\right)\right]. \tag{57}$$

where $(i,i') = \{(H,F),(F,H)\}$*, that is, these value functions are characterized for links between Home and Foreign firms, both in import and export activity.*

Since in this version of the model production networks are endogenous, one can endogenously define the total resources used to pay for the fixed costs in forming relationships between Home and Foreign firms

$$\begin{aligned} L_{Ft}^{F} &= \int\int \nu_F m_{Ft-1}(\phi_F,\phi_H)\delta_F\mathbb{E}_t\left(\epsilon_{Ft}\right)dG_{\phi_F}(\phi_F)dG_{\phi_H}(\phi_H) \\ &+ \int\int (1-\nu_F)a_{Ft}(\phi_F,\phi_H)\delta_F\mathbb{E}_t\left(\epsilon_{Ft}|\epsilon_{Ft}<\bar{\epsilon}_{Ft}(\phi_F,\phi_H)\right)dG_{\phi_F}(\phi_F)dG_{\phi_H}(\phi_H). \end{aligned}$$

As with (26), the first integral represents the fixed costs paid by relationships that did not have the chance to be reevaluated, weighted by the mass of those, evaluated at the average fixed cost (since there is no selection on who has to pay for that group), which is $\delta_F = \mathbb{E}_t(\epsilon_{Ft})$. The second integral represents the fixed cost paid by relationships that had the chance to be reevaluated and were accepted. Note that since the seller is the active side in the formation of relationships, Home only pays the fixed costs of forming relationships in export activity. The fixed costs of forming relationships in import activity are paid by Foreign.

The expectation of the random component of relationships' fixed cost conditional on the relationship being active and its corresponding threshold for activating and terminating relationships between Home and Foreign firms are

$$\begin{aligned} \mathbb{E}_t\left(\epsilon_{Ft}|\epsilon_{Ft}<\bar{\epsilon}_{Ft}(\phi_i,\phi'_{i'})\right) &= \int_0^{\bar{\epsilon}_{Ft}(\phi_i,\phi'_{i'})}\epsilon_{Ft}dG_{\epsilon_F}(\epsilon_F), \\ \bar{\epsilon}_{Ft}(\phi_i,\phi'_{i'}) &= \max\left\{\frac{\pi_{Ft}(\phi_i,\phi'_{i'})}{\delta_F} + \sum_{k=1}^{\infty}(\beta\nu_F)^k\left(\frac{\pi_{Ft+k}(\phi_i,\phi'_{i'})}{\delta_F} - 1\right), 0\right\}, \end{aligned}$$

where $(i,i') = \{(H,F),(F,H)\}$.

B.3 Network Connectivity

To understand how the structure of production in the production network generates connectivity between firms, take the marginal cost of a firm ϕ_H, $c(\phi_H)$, and iterate across upstream firms, both

direct and indirect suppliers:[64]

$$\begin{aligned}
c_H(\phi_H)^{1-\sigma} &= \left(\alpha^L\right)^{\sigma}\left(\phi_H^L\right)^{\sigma-1}+\left(1-\alpha^L\right)^{\sigma}\mu^{1-\sigma}\int\tau_F^{1-\sigma}m_F(\phi_H,\phi_F)\left(\phi_{HF}^P\right)^{\sigma-1}dG_{\phi_F}(\phi_F)\\
&\quad+\left(1-\alpha^L\right)^{\sigma}\mu^{1-\sigma}\int m_H(\phi_H,\phi_H')c(\phi_H')^{1-\sigma}dG_{\phi_H}(\phi_H'),\\
&= \bar{\phi}^P(\phi_H)^{\sigma-1}+\gamma^c\int m_H(\phi_H,\phi_H')\bar{\phi}^P(\phi_H')^{\sigma-1}dG_{\phi_H}(\phi_H')\\
&\quad+\int\int\left(\gamma^c\right)^2m_H(\phi_H,\phi_H')m_H(\phi_H',\phi_H'')c(\phi_H'')^{1-\sigma}dG_{\phi_H}(\phi_H')dG_{\phi_H}(\phi_H''),\\
&= \bar{\phi}^P(\phi_H)^{\sigma-1}+\gamma^c\int m_H(\phi_H,\phi_H')\bar{\phi}^P(\phi_H')^{\sigma-1}dG_{\phi_H}(\phi_H')\\
&\quad+(\gamma^c)^2\int\int m_H(\phi_H,\phi_H')m_H(\phi_H',\phi_H'')\bar{\phi}^P(\phi_H'')^{\sigma-1}dG_{\phi_H}(\phi_H')dG_{\phi_H}(\phi_H'')\\
&\quad+\int\int\int\left(\gamma^c\right)^3m_H(\phi_H,\phi_H')m_H(\phi_H',\phi_H'')m_H(\phi_H'',\phi_H''')\times\\
&\quad c(\phi_H''')^{1-\sigma}dG_{\phi_H}(\phi_H')dG_{\phi_H}(\phi_H'')dG_{\phi_H}(\phi_H'''),
\end{aligned}$$

where $\bar{\phi}^P(\phi_H)^{\sigma-1}=\left(\alpha^L\right)^{\sigma}\left(\phi_H^L\right)^{\sigma-1}+\left(1-\alpha^L\right)^{\sigma}\mu^{1-\sigma}\int\tau_F^{1-\sigma}m_F(\phi_H,\phi_F)\left(\phi_{HF}^P\right)^{\sigma-1}dG_{\phi_F}(\phi_F)$ is a measure of ϕ_H's productivity across her own labor-augmenting productivity and the productivity of foreign inputs, $\mu=\frac{\sigma}{\sigma-1}$ is the constant markup and $\gamma^c=\mu\left(\frac{1-\alpha^L}{\mu}\right)^{\sigma}$ is the decay parameter of the network. If $\gamma^c<1$,[65] then in the limit one has the following

$$c_H(\phi_H)^{1-\sigma} = \int\underbrace{\left[\sum_{d=0}^{\infty}\left(\gamma^c\right)^d g^{[d]}(\phi_H,\phi_H')\right]}_{h(\phi_H,\phi_H';\gamma^c)}\left(\frac{1}{\bar{\phi}^P(\phi_H')}\right)^{1-\sigma}dG_{\phi_H}(\phi_H'),$$

where

$$\begin{aligned}
g^{[0]}(\phi_H,\phi_H') &= \begin{cases}1\ if\ \phi_H=\phi_H'\\ 0\ otherwise\end{cases},\\
g^{[1]}(\phi_H,\phi_H') &= m_H(\phi_H,\phi_H')g_{\phi_H}(\phi_H'),\\
g^{[2]}(\phi_H,\phi_H') &= \int m_H(\phi_H,\phi_H'')g_{\phi_H}(\phi_H'')m_H(\phi_H'',\phi_H')g_{\phi_H}(\phi_H')d\phi_H'',\\
g^{[3]}(\phi_H,\phi_H') &= \int\int m_H(\phi_H,\phi_H'')g_{\phi_H}(\phi_H'')m_H(\phi_H'',\phi_H''')g_{\phi_H}(\phi_H''')m_H(\phi_H''',\phi_H')g_{\phi_H}(\phi_H')d\phi_H'''d\phi_H'',\\
&\vdots\\
g^{[d]}(\phi_H,\phi_H') &= \int g^{[d-1]}(\phi_H,\phi_H'')g_{\phi_H}(\phi_H'')m_H(\phi_H'',\phi_H')g_{\phi_H}(\phi_H')d\phi_H''.
\end{aligned}$$

[64] In order to obtain closed-form solutions and thus gain more intuition, assume that all elasticities of substitution are equal: $\sigma_H^G=\sigma_F^G=\sigma_{FH}^P=\sigma_{HH}^P=\sigma_{HF}^P=\sigma^X=\sigma^L=\sigma$ and $(1-\alpha^L)^{\sigma}<\mu^{\sigma-1}$.

[65] Given the market structure of monopolistic competition, one needs $\sigma>1$, which in turn implies that $\gamma^c<1$. Nevertheless, although $\sigma>1$ is sufficient, it is not necessary: $\gamma^c<1\Leftrightarrow(1-\alpha^L)^{\sigma}<\mu^{\sigma-1}$.

The function $g^{[d]}(\phi_H, \phi'_H)$ measures all direct and indirect paths of length $d \geq 0$ between ϕ_H and ϕ'_H. Thus, $h(\phi_H, \phi'_H; \gamma^e) = \sum_{d=0}^{\infty} (\gamma^e)^d \, g^{[d]}(\phi_H, \phi'_H)$ measures the connectivity between ϕ_H and ϕ'_H. Moreover, it measures the total mass of paths between ϕ_H and ϕ'_H, for paths of all possible lengths $d \geq 0$, where each path is weighted by $(\gamma^e)^d$. Note that the weight of each path, $(\gamma^e)^d$, is increasing in the intensity of intermediate inputs, $1 - \alpha^L$, and decreasing in markups, μ. In other words, the more intensively firms use intermediate inputs and the lower markups are, the more distant paths between firms will affect any given firm in the economy, i.e., the more connected firms are.

B.4 Proofs

Proof of Proposition 1. Case 1 is straightforward from the definition of Home firms' value functions in (19), (20) and (21). For case 3, one computes $V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H)$ and iterates forward to obtain (24):

$$\begin{aligned} V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H) &= \pi^P_{Ht}(\phi_H, \phi'_H) - \delta_H \epsilon_{Ht} \\ &\quad + \beta\nu_H \mathbb{E}_t \left[V^A_{Ht+1}(\phi_H, \phi'_H|\epsilon_{Ht+1}) - V^I_{Ht+1}(\phi_H, \phi'_H) \right], \\ V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H) &= \pi^P_{Ht}(\phi_H, \phi'_H) - \delta_H \epsilon_{Ht} \\ &\quad + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left[\pi^P_{Ht+k}(\phi_H, \phi'_H) - \delta_H \right]. \end{aligned}$$

Then, one can get (25) with the following

$$\begin{aligned} Pr\left[V^A_{Ht}(\phi_H, \phi'_H|\epsilon_{Ht}) - V^I_{Ht}(\phi_H, \phi'_H) > 0 \right] &= Pr\left[\frac{\pi^P_{Ht}(\phi_H, \phi'_H)}{\delta_H} + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left(\frac{\pi^P_{Ht+k}(\phi_H, \phi'_H)}{\delta_H} - 1 \right) > 0 \right], \\ &= G_{\epsilon_H}\left[\frac{\pi^P_{Ht}(\phi_H, \phi'_H)}{\delta_H} + \sum_{k=1}^{\infty} (\beta\nu_H)^k \left(\frac{\pi^P_{Ht+k}(\phi_H, \phi'_H)}{\delta_H} - 1 \right) \right]. \end{aligned}$$

Finally, case 2 can be obtained by setting π^P_{Ht}, V^R_{Ht}, V^A_{Ht} and V^I_{Ht} constant in (24).

■

Proof of Proposition 2. Start from a type ϕ_H cost minimization solution and iterate across upstream firms as in Section B.3:

$$
\begin{aligned}
c_H(\phi_H)^{1-\sigma} &= \left(\alpha^L\right)^{\sigma}\left(\phi_H^L\right)^{\sigma-1} + \left(1-\alpha^L\right)^{\sigma}\mu^{1-\sigma}\int \tau_F^{1-\sigma} m_F(\phi_H,\phi_F)\left(\phi_{HF}^P\right)^{\sigma-1} dG_{\phi_F}(\phi_F) \\
&\quad + \left(1-\alpha^L\right)^{\sigma}\mu^{1-\sigma}\int m_H(\phi_H,\phi_H')c(\phi_H')^{1-\sigma} dG_{\phi_H}(\phi_H'), \\
&= \bar{\phi}^P(\phi_H)^{\sigma-1} + \gamma^c \int m_H(\phi_H,\phi_H')\bar{\phi}^P(\phi_H')^{\sigma-1} dG_{\phi_H}(\phi_H') \\
&\quad + \int\int \left(\gamma^c\right)^2 m_H(\phi_H,\phi_H')m_H(\phi_H',\phi_H'')c(\phi_H'')^{1-\sigma} dG_{\phi_H}(\phi_H')dG_{\phi_H}(\phi_H''), \\
&= \int \left[\sum_{d=0}^{\infty}\left(\gamma^c\right)^d g^{[d]}(\phi_H,\phi_H')\right]\left(\frac{1}{\bar{\phi}^P(\phi_H')}\right)^{1-\sigma} dG_{\phi_H}(\phi_H'), \\
&= \int h(\phi_H,\phi_H';\gamma^c)\left(\frac{1}{\bar{\phi}^P(\phi_H')}\right)^{1-\sigma} dG_{\phi_H}(\phi_H'),
\end{aligned}
$$

where $\gamma^c < 1$. Similarly for $y(\phi_H)$, start from type ϕ_H output clearing condition (15), replace with the optimal solutions of (38), (40) and (41), and then iterate across downstream firms:

$$
\begin{aligned}
y(\phi_H) &= \left(\phi_H^C\right)^{\sigma-1}\left(\mu c_H(\phi_H)\right)^{-\sigma} D_H \\
&\quad + \int m_F(\phi_F,\phi_H)\left(\phi_{FH}^P\phi_F^C\right)^{\sigma-1}\left(\mu\tau_F c_H(\phi_H)\right)^{-\sigma} D_F dG_{\phi_F}(\phi_F) \\
&\quad + \int m_H(\phi_H',\phi_H)\left(1-\alpha^L\right)^{\sigma}\left(\mu c_H(\phi_H)\right)^{-\sigma} c_H(\phi_H')^{\sigma} y(\phi_H') dG_{\phi_H}(\phi_H'), \\
\frac{c_H(\phi_H)^{\sigma}y(\phi_H)}{D_H} &= \left(\phi_H^C\right)^{\sigma-1}\mu^{-\sigma} + \mu^{-\sigma}\int m_F(\phi_F,\phi_H)\left(\phi_{FH}^P\phi_F^C\right)^{\sigma-1}\tau_F^{-\sigma}\frac{D_F}{D_H} dG_{\phi_F}(\phi_F) \\
&\quad + \left(1-\alpha^L\right)^{\sigma}\mu^{-\sigma}\int m_H(\phi_H',\phi_H)\frac{c_H(\phi_H')^{\sigma}y(\phi_H')}{D_H} dG_{\phi_H}(\phi_H'), \\
\frac{c_H(\phi_H)^{\sigma}y(\phi_H)}{D_H} &= \mu^{-\sigma}\left(\bar{\phi}^C(\phi_H)\right)^{\sigma-1} + \left(1-\alpha^L\right)^{\sigma}\mu^{-\sigma}\int m_H(\phi_H',\phi_H)\frac{c_H(\phi_H')^{\sigma}y(\phi_H')}{D_H} dG_{\phi_H}(\phi_H'), \\
&\vdots \\
\frac{y(\phi_H)}{D_H} &= c_H(\phi_H)^{-\sigma}\mu^{-\sigma}\int h(\phi_H',\phi_H;\gamma^y)\left(\bar{\phi}^C(\phi_H')\right)^{\sigma-1} dG_{\phi_H}(\phi_H'),
\end{aligned}
$$

where I use the optimal pricing from (45)-(47), and $\bar{\phi}^C(\phi_H)$ is the quality shifter index of firm ϕ_H across domestic and foreign demand sources.

■

Proof of Proposition 3. Combine the definition of D_H with $Q_H = E_H/P_H$ to obtain $Q_H = D_H/\left(P_H\right)^{\sigma}$. Use the labor market clearing condition (14) together with the solution to $y(\phi_H)$ in Proposition 2 to obtain an expression for D_H:

$$
\begin{aligned}
L - L_H^F - L_F^F &= \int \left(\alpha^L\right)^{\sigma} \left(\phi_H^L\right)^{\sigma-1} \text{¢}_H(\phi_H)^{\sigma} y(\phi_H) dG_{\phi_H}(\phi_H), \\
&= D_H \mu^{-\sigma} \left(\alpha^L\right)^{\sigma} \int \left(\phi_H^L\right)^{\sigma-1} \left[\int h(\phi'_H, \phi_H; \gamma^y) \left(\bar{\phi}^C(\phi'_H)\right)^{\sigma-1} dG_{\phi_H}(\phi'_H)\right] dG_{\phi_H}(\phi_H), \\
\Rightarrow D_H &= \mu^{\sigma}(L - L_H^F - L_F^F) \left[\left(\alpha^L\right)^{\sigma} \int\int h(\phi'_H, \phi_H; \gamma^y) \left(\bar{\phi}^C(\phi'_H)\phi_H^L\right)^{\sigma-1} dG_{\phi_H}(\phi'_H) dG_{\phi_H}(\phi_H)\right]^{-1}.
\end{aligned}
$$

Use the definition of the ideal price index P_H from (43) and the optimal pricing of $p_H^C(\phi_H)$ from (47) together with the solution to $\text{¢}_H(\phi_H)$ in Proposition 2 to obtain an expression for P_H:

$$
\begin{aligned}
P_H^{\sigma} &= \mu^{\sigma} \left[\int \left(\phi_H^C\right)^{\sigma-1} \text{¢}_H(\phi_H)^{1-\sigma} dG_{\phi_H}(\phi_H)\right]^{\frac{\sigma}{1-\sigma}}, \\
&= \mu^{\sigma} \left[\int\int h(\phi_H, \phi'_H; \gamma^{\text{¢}}) \left(\bar{\phi}^P(\phi'_H)\phi_H^C\right)^{\sigma-1} dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H)\right]^{\frac{\sigma}{1-\sigma}}.
\end{aligned}
$$

Putting the solution for D_H and P_H I get the result:

$$
\begin{aligned}
Q_H &= (L - L_H^F - L_F^F) \frac{\left[\int\int h(\phi_H, \phi'_H; \gamma^{\text{¢}}) \left(\bar{\phi}^P(\phi'_H)\phi_H^C\right)^{\sigma-1} dG_{\phi_H}(\phi_H) dG_{\phi_H}(\phi'_H)\right]^{\frac{\sigma}{1-\sigma}}}{(\alpha^L)^{\sigma} \int\int h(\phi'_H, \phi_H; \gamma^y) \left(\bar{\phi}^C(\phi'_H)\phi_H^L\right)^{\sigma-1} dG_{\phi_H}(\phi'_H) dG_{\phi_H}(\phi_H)}, \\
&= (L - L_H^F - L_F^F) \underbrace{\frac{\left[\int\int h(\phi'_H, \phi_H; \gamma^y) \left(\phi_H^{C\prime} \bar{\phi}^P(\phi_H)\right)^{\sigma-1} dG_{\phi_H}(\phi'_H) dG_{\phi_H}(\phi_H)\right]^{\frac{\sigma}{1-\sigma}}}{\int\int h(\phi'_H, \phi_H; \gamma^y) \left(\bar{\phi}^C(\phi'_H)\phi_H^L\right)^{\sigma-1} dG_{\phi_H}(\phi'_H) dG_{\phi_H}(\phi_H)}}_{\text{¢}_H}.
\end{aligned}
$$

■

Proof of Proposition 4. View proof of Proposition 1. ■

B.5 Conditional Mean of Weibull Distribution

In this appendix I show that the conditional mean of the Weibull distribution has a simple and closed form solution. This is important in order to solve for the dynamic problem of firms' production links creation and destruction (e.g. see Equations (26) and (27)). The density of the Weibull distribution is the following:

$$
f_W(x; \lambda, s_x) = \begin{cases} \frac{s_x}{\lambda} \left(\frac{x}{\lambda}\right)^{s_x - 1} e^{-\left(\frac{x}{\lambda}\right)^{s_x}} & x \geq 0 \\ 0 & x < 0 \end{cases},
$$

where λ and s_x are the scale and shape parameters, respectively. Define the conditional mean $\overline{x}_a^b = \int_a^b x f_W(x) dx$. With a simple change of variables $u = \left(\frac{x}{\lambda}\right)^{s_x}$, it is straightforward to see

that $\overline{x}_a^b = \lambda \int_a^b u^{\frac{1}{s_x}} e^u du$. Given that the lower incomplete gamma function is defined as: $\gamma(s,b) = \int_0^b u^{s-1}e^{-u}du$ and the upper incomplete gamma function as $\Gamma(s,a) = \int_a^\infty u^{s-1}e^{-u}du$, one has the following:

$$\overline{x}_0^b = \lambda\gamma\left(1+\frac{1}{s_x}, b\right),$$
$$\overline{x}_a^\infty = \lambda\Gamma\left(1+\frac{1}{s_x}, a\right).$$

Appendix C Algorithms for Simulating the Model

This appendix describes how the model is simulated, both in the steady state and in the transition path between steady states.

C.1 Simulation of the Steady State

The steps for simulating the model's steady state are the following:

1. Start with a guess for $c_H(\phi_H)$, $y(\phi_H)$, $p_H^{X,G}(\phi_H)$, $p_F^{X,G}(\phi_H)$, D_H and D_F.[66]
2. Given the solutions in (32)-(47), compute bilateral gross profits $\pi_i(\phi_i, \phi'_{i'})$ using (48) and the acceptance functions $a_i(\phi_i, \phi'_{i'})$ using (23) and (56).
3. Given that in the steady state the acceptance function is equal to the matching function, compute $\hat{p}_i^{X,G}(\phi_H)$ using (34) and new versions of price indeces and quantities from (32)-(47).[67]
4. Compute $\hat{c}_H(\phi_H)$ and $\hat{y}(\phi_H)$ using (32) and (15), respectively.
5. Compute net profits using (50) and thus aggregate income of Home's representative household. Use (13), (44) and the fact that the aggregate trade surplus is assumed to be a constant fraction of aggregate income to compute $\hat{D}_H$.
6. Compute export flows using (41) and (46), and import flows using (38) and (45). Then compute $\hat{D}_F$ using (12).
7. Compute the distances

[66] $p_i^{X,G}(\cdot)$ can be excluded from the iteration of the algorithm if $\sigma_{Hi}^P = \sigma^X$. The purpose of adding $p_i^{X,G}(\phi_H)$ to the iteration when $\sigma_{Hi}^P \neq \sigma^X$ is to avoid iterating over $m_i(\cdot,\cdot)$.

[67] Note that Home's acceptance function, matching function and bilateral profits are $N_H \times N_H$ matrices, where N_H is the number of grids of the space S_{ϕ_H}. To reduce computational costs, I implement these matrices using sparsity techniques. These techniques reduce the information and size of these matrices significantly, reducing in turn the time the algorithm takes to converge.

$$
\begin{aligned}
d_{SS}^{c} &\equiv \max_{\phi_H \in S_{\phi_H}} \left\{ \left| c_H(\phi_H) - \hat{c}_H(\phi_H) \right| \right\}, \\
d_{SS}^{y} &\equiv \max_{\phi_H \in S_{\phi_H}} \left\{ \left| y_H(\phi_H) - \hat{y}_H(\phi_H) \right| \right\}, \\
d_{SS}^{p_i} &\equiv \max_{\phi_H \in S_{\phi_H}} \left\{ \left| p_i^{X,G}(\phi_H) - \hat{p}_i^{X,G}(\phi_H) \right| \right\}, \ i = \{H, F\}, \\
d_{SS} &\equiv \max \left\{ d_{SS}^{c}, d_{SS}^{y}, d_{SS}^{p_H}, d_{SS}^{p_F}, \left| D_H - \hat{D}_H \right|, \left| D_F - \hat{D}_F \right| \right\}.
\end{aligned}
$$

If $d_{SS} \leq \epsilon$ for a arbitrary small ϵ, convergence has been achieved and the algorithm is over. If $d_{SS} > \epsilon$, update $c_H(\phi_H)$, $y(\phi_H)$, $p_H^{X,G}(\phi_H)$, $p_F^{X,G}(\phi_H)$, D_H and D_F according to

$$
\begin{aligned}
c_H(\phi_H) &= \omega \hat{c}_H(\phi_H) + (1-\omega) c_H(\phi_H), \\
y_H(\phi_H) &= \omega \hat{y}_H(\phi_H) + (1-\omega) y_H(\phi_H) \\
p_i^{X,G}(\phi_H) &= \omega \hat{p}_i^{X,G}(\phi_H) + (1-\omega) p_i^{X,G}(\phi_H), \ i = \{H, F\}, \\
D_i &= \omega \hat{D}_i + (1-\omega) D_i, \ i = \{H, F\},
\end{aligned}
$$

where $\omega \in (0, 1)$. Then return to step 2 and iterate until convergence.

The lack of a proof of uniqueness in the model with endogenous networks means that convergence is not guaranteed. Nevertheless, starting from different initial guesses as a robustness helps to evaluate how sensitive the solution is. This robustness test was implemented and passed for a wide range of parameter values, downgrading the concern of multiplicity of the steady state.

A number of suggestions can be implemented in order to speed up this algorithm. First, step 2 and therefore requires computing large matrices which can be implemented by using sparse matrix techniques, since the bilateral profit,[68] matching and acceptance matrix are highly sparse. Second, step 5 requires to compute the fixed costs paid in forming relationships. This, in turn, requires to compute the conditional expectation of ϵ_{it}. Rather than integrating, one can use the closed-form solutions from Appendix B.5.

C.2 Simulation of the Transition Path between Steady States

The challenge of computing the transition path between steady states is to compute the matching functions at any t, given the matching functions at $t-1$. In order to do so, one needs to know the future bilateral gross profits for all potential links in the economy, which in turn requires to know how many periods the transition path takes. Thus, besides solving the equilibrium for any t, one needs to iterate on the full path of bilateral gross profits and the number of periods the transition path takes. The steps for simulating the model's transition path between an initial steady state in $t = 0$ and a final steady state in $t = T$ are the following:

1. Start with a guess of how many periods it will take until the new steady state, $\hat{T}$.

[68] After imputing the negative values of profits with zeros.

2. Make a guess of the bilateral gross profit function path $\{\hat{\pi}_{it}(\cdot,\cdot)\}_{t=1}^{\hat{T}}$. For example, start with the average between the initial and final steady state: $\hat{\pi}_{it}(\phi_i,\phi'_{i'}) = \frac{1}{2}\left(\pi_{i0}(\phi_i,\phi'_{i'}) + \pi_{iT}(\phi_i,\phi'_{i'})\right)$ for $(i,i') = \{(H,H),(H,F),(F,H)\}$.

3. In each period $t = \{1,\ldots,\hat{T}\}$, given $m_{it-1}(\phi_i,\phi'_{i'})$, solve for the equilibrium:

   (a) Make a guess for $\text{-}c_{Ht}(\phi_H)$, $y_t(\phi_H)$, $p_{Ht}^{X,G}(\phi_H)$, $p_{Ft}^{X,G}(\phi_H)$, D_{Ht} and D_{Ft}.

   (b) Given the solutions in (32)-(47), compute bilateral gross profits $\pi_{it}(\phi_i,\phi'_{i'})$ using (48). Compute the acceptance functions $a_{it}(\phi_i,\phi'_{i'})$ using (25) and (57). In order to do this, set $\pi_{it+s}(\phi_i,\phi'_{i'}) = \hat{\pi}_{it+s}(\phi_i,\phi'_{i'})$ for $s = \{1,\ldots,\hat{T}-t\}$ and $\pi_{it+s}(\phi_i,\phi'_{i'}) = \pi_{iT}(\phi_i,\phi'_{i'})$ for $s > \hat{T}-t$.

   (c) Compute the matching functions using (17) and (18). Use these to compute $\hat{p}_{it}^{X,G}(\phi_H)$ using (34) and new versions of price indexes and quantities from (32)-(47).[69]

   (d) Compute $\hat{\text{-}c}_{Ht}(\phi_H)$ and $\hat{y}_t(\phi_H)$ using (32) and (15), respectively.

   (e) Compute net profits using (50) and thus aggregate income of Home's representative household. Use (13), (44) and the fact that the aggregate trade surplus is assumed to be a constant fraction of aggregate income to compute $\hat{D}_{Ht}$.

   (f) Compute export flows using (41) and (46), and import flows using (38) and (45). Then compute $\hat{D}_{Ft}$ using (12).

   (g) Compute the distances

$$
\begin{aligned}
d_t^{c} &\equiv \max_{\phi_H \in S_{\phi_H}} \left\{ \left| \text{-}c_{Ht}(\phi_H) - \hat{\text{-}c}_{Ht}(\phi_H) \right| \right\}, \\
d_t^{y} &\equiv \max_{\phi_H \in S_{\phi_H}} \left\{ \left| y_{Ht}(\phi_H) - \hat{y}_{Ht}(\phi_H) \right| \right\}, \\
d_t^{p_i} &\equiv \max_{\phi_i \in S_{\phi_i}} \left\{ \left| p_{it}^{X,G}(\phi_i) - \hat{p}_{it}^{X,G}(\phi_i) \right| \right\}, \; i = \{H,F\}, \\
d_t &\equiv \max \left\{ d_t^{c}, d_t^{y}, d_t^{p_H}, d_t^{p_F}, \left| D_{Ht} - \hat{D}_{Ht} \right|, \left| D_{Ft} - \hat{D}_{Ft} \right| \right\}.
\end{aligned}
$$

   If $d_t \leq \epsilon$ for a arbitrary small ϵ, convergence of the equilibrium has been achieved and the algorithm can be continued. If $d_t > \epsilon$, update $\text{-}c_{Ht}(\phi_H)$, $y_t(\phi_H)$, $p_{Ht}^{X,G}(\phi_H)$, $p_{Ft}^{X,G}(\phi_H)$, D_{Ht} and D_{Ft} according to

$$
\begin{aligned}
\text{-}c_{Ht}(\phi_H) &= \omega \hat{\text{-}c}_{Ht}(\phi_H) + (1-\omega)\text{-}c_{Ht}(\phi_H), \\
y_{Ht}(\phi_H) &= \omega \hat{y}_{Ht}(\phi_H) + (1-\omega) y_{Ht}(\phi_H), \\
p_{it}^{X,G}(\phi_H) &= \omega \hat{p}_{it}^{X,G}(\phi_H) + (1-\omega) p_{it}^{X,G}(\phi_H), \; i = \{H,F\}, \\
D_{it} &= \omega \hat{D}_{it} + (1-\omega) D_{it}, \; i = \{H,F\},
\end{aligned}
$$

   where $\omega \in (0,1)$. Then return to step 3.(b) and iterate until convergence.

[69] As with the steady state, I use sparse matrix techniques to reduce the computational costs of the algorithm.

4. Compute the distance

$$d^{\pi} \equiv \max_{t=\{1,...,\hat{T}\}} \max_{(\phi_i,\phi'_{i'})\in(S_{\phi_i}\times S_{\phi_{i'}})} \left\{\left|\pi_{it}(\phi_i,\phi'_{i'}) - \hat{\pi}_{it}(\phi_i,\phi'_{i'})\right|\right\}, \ (i,i') = \{(H,H),(H,F),(F,H)\}.$$

If $d^{\pi} \leq \epsilon$ for a arbitrary small ϵ, convergence of the equilibrium has been achieved and the algorithm can be continued. If $d^{\pi} > \epsilon$, update $\{\pi_{it}(\phi_i,\phi'_{i'})\}_{t=1}^{\hat{T}}$ according to

$$\pi_{it}(\phi_i,\phi'_{i'}) = \omega\hat{\pi}_{it}(\phi_i,\phi'_{i'}) + (1-\omega)\pi_{it}(\phi_i,\phi'_{i'}), \ (i,i') = \{(H,H),(H,F),(F,H)\},$$

where $\omega \in (0,1)$. Then return to step 2 and iterate until convergence.

5. Compute the distance

$$d^{m} \equiv \max_{(\phi_i,\phi'_{i'})\in(S_{\phi_i}\times S_{\phi_{i'}})} \left\{\left|m_{iT}(\phi_i,\phi'_{i'}) - m_{i\hat{T}}(\phi_i,\phi'_{i'})\right|\right\}, \ (i,i') = \{(H,H),(H,F),(F,H)\}.$$

If $d^{m} \leq \epsilon$ for a arbitrary small ϵ, convergence of the transition path has been achieved and the algorithm is over. If $d^{m} > \epsilon$, update $\hat{T} = \hat{T} + 1$ and return to step 1 and iterate until convergence.

As with the steady state, the lack of a proof of uniqueness in the model with endogenous networks means that convergence is not guaranteed. Nevertheless, starting from different initial guesses as a robustness helps to evaluate how sensitive the solution is. This robustness test was implemented and passed for a wide range of parameter values, downgrading the concern of multiplicity of the transition path.

A number of suggestions can be implemented in order to speed up this algorithm. First, guesses made in step 3 can be made by using the solution of a previous iteration. In the case of the first iteration and with $t = 1$, one can use the initial steady state as a first guess. Second, guesses made in step 2 can made by using the solution of a previous iteration (this only works after the first iteration, where the suggestion made in the algorithm can be a first guess). Third, step 3 is similar to the algorithm that simulates the steady state. The difference is that computing the matching functions is computationally more challenging since one needs to compute the present discounted value of each link. This can be implemented by using high-dimensional sparse matrix techniques, since the bilateral profit matrix is highly sparse.[70] Fourth, step 5 requires to compute the fixed costs paid in forming relationships. This in turn, requires to compute the conditional expectation of ϵ_{it}. Rather than integrating, one can use the closed-form solutions from Appendix B.5.

[70] After imputing the negative values of the matrix with zeros.

Appendix D Structural Estimation

D.1 Details of the SMM

Given that the model does not have closed-form solutions for all the parameters involved in the mapping between data, parameters and primitives, I use a standard SMM technique to estimate the parameters using a set of micro moments from both the cross-section and dynamics of firm behavior in Chile. The algorithm proceeds in four steps. In the first, given a value for Θ, the model is simulated using a procedure described in Appendix C.1. In a second step, using the simulation of the model, a set of moments is produced and stacked into the vector $\hat{f}(\Theta)$. In the third step, the same set of moments is produced with the data and stacked into the vector f. Finally, an objective function is computed to evaluate the deviations of the simulated moments from the data moments, $d_{SMM}(\Theta) = f - \hat{f}(\Theta)$. If this difference is not minimized according to some threshold, the algorithm is repeated for a different set of parameter values, until a minimum is reached. The estimation procedure is based on the following moment condition:

$$\mathbb{E}\left[d_{SMM}(\Theta_0)\right] = 0$$

where Θ_0 is the true value of Θ. Thus, the algorithm looks for $\hat{\Theta}$ such that

$$\hat{\Theta} = \arg\min_{\Theta}\{d_{SMM}(\Theta)'\mathbf{W}d_{SMM}(\Theta)\}$$

where $\mathbf{W}$ is a weighting matrix which is the generalized inverse of the estimated variance-covariance matrix of the moments calculated from the data. For now, I assume the identity matrix, which effectively weights all the moments equally.

Although the dimensionality of Θ is large, I describe a set of steps that reduce the computation time of iterating in the SMM. These steps leverage predictions from the model that exploit dependencies between parameters. First, I show how to derive Foreign's primitives ϕ_P and the iceberg trade cost τ_F in Appendix D.4. These primitives can be computed directly given data and the elasticities of substitution and thus, so can the parameters governing F_{ϕ_F} and F_{τ_F} be computed. Second, I use the following relationships between CES weights and elasticities of substitution:

$$1 - \alpha^L = \frac{p^{X,T}(\phi_H)}{c_H(\phi_H)}\left(\frac{x^T(\phi_H)}{y(\phi_H)}\right)^{\frac{1}{\sigma^L}}, \tag{58}$$

$$\alpha^I = \frac{p_F^{X,G}(\phi_H)}{p^{X,T}(\phi_H)}\left(\frac{x_F^G(\phi_H)}{y(\phi_H)}\right)^{\frac{1}{\sigma^X}}. \tag{59}$$

This implies that, given values of σ^L, σ^X, and simulation of the characteristics in (58) and (59), one does not need to iterate on α^L and α^I.[71] Third, conditional on the previous step and on values for the elasticities of substitution, one can use (39) and (40) to compute $\phi_H = (\phi_H^L, \phi_H^C, \phi_F^C)$ and

[71] For the initial conditions, I compute α^L and α^I directly from the data by setting $\sigma^L = 1$ and $\sigma^X = 1$.

thus compute the parameters from F_{ϕ_H}.[72] These three steps reduce the dimensionality of Θ from 28 to 16. I use the Particle Swarm algorithm to implement the SMM.

D.2 Model's Moments for SMM

I describe how to produce moments from firms' production network dynamics using the model in the steady state. These moments are used in the SMM procedure. First, I produce the number of links firms have:

$$\begin{aligned} N_H^{S,i}(\phi_H) &= \int \mathbb{1}\{m_H(\phi_H, \phi_i') > 0\} dF_{\phi_i}(\phi_i'),\ i = \{H, F\}, \\ N_H^{B,i}(\phi_H) &= \int \mathbb{1}\{m_H(\phi_i', \phi_H) > 0\} dF_{\phi_i}(\phi_i'),\ i = \{H, F\}, \end{aligned}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function equal to 1 if the expression in $\{\cdot\}$ is true and 0 otherwise. $N_H^{S,i}(\phi_H)$ and $N_H^{B,i}(\phi_H)$ measure the number of types that ϕ_H supplies and buys from country i, respectively. These measures are compared in the SMM with the actual number of links that firms have.

Second, I produce survival and destruction rates of Home and Foreign suppliers that firms have:

$$\begin{aligned} s_H^{S,i}(\phi_H) &= \frac{\int \left(\nu_i m_H(\phi_H, \phi_i') + (1-\nu_i) m_H(\phi_H, \phi_i') a_H(\phi_H, \phi_i')\right) dF_{\phi_i}(\phi_i')}{\int m_H(\phi_H, \phi_i') dF_{\phi_i}(\phi_i')},\ i = \{H, F\}, \\ d_H^{S,i}(\phi_H) &= \frac{\int (1-\nu_i)(1 - a_H(\phi_H, \phi_i')) m_H(\phi_H, \phi_i') dF_{\phi_i}(\phi_i')}{\int m_H(\phi_H, \phi_i') dF_{\phi_i}(\phi_i')},\ i = \{H, F\}, \end{aligned}$$

where $s_H^{S,i}(\phi_H)$ and $d_H^{S,i}(\phi_H)$ is the fraction of suppliers from country i that ϕ_H keeps and destroys from one period to the next, respectively. Similar objects can be produce relative to buyers:

$$\begin{aligned} s_H^{B,i}(\phi_H) &= \frac{\int \left(\nu_i m_H(\phi_i', \phi_H) + (1-\nu_i) m_H(\phi_i', \phi_H) a_H(\phi_i', \phi_H)\right) dF_{\phi_i}(\phi_i')}{\int m_H(\phi_i', \phi_H) dF_{\phi_i}(\phi_i')},\ i = \{H, F\}, \\ d_H^{B,i}(\phi_H) &= \frac{\int (1-\nu_i)(1 - a_H(\phi_i', \phi_H)) m_H(\phi_i', \phi_H) dF_{\phi_i}(\phi_i')}{\int m_H(\phi_i', \phi_H) dF_{\phi_i}(\phi_i')},\ i = \{H, F\}, \end{aligned}$$

where $s_H^{B,i}(\phi_H)$ and $d_H^{B,i}(\phi_H)$ is the fraction of buyers from country i that ϕ_H keeps and destroys from one period to the next, respectively. These moments are produced with the structure from the data and compared in the SMM procedure.

[72] For the initial conditions, I take a random draw of the parameters from F_{ϕ_H}.

Table D.5: Model's Fit to Moments of Firm Size and International Flows and Prices

Moment	Related Parameter	Model	Data
Var. Firms' Domestic Final Sales	$v(\phi_H^C)$	0.83	0.62
Var. Firms' Exports Productivity	$v(\phi_{FH}^P)$	0.82	0.97
Var. Firms' Employment	$v(\phi_H^L)$	0.95	0.92
Cov. Firms' Domestic and Foreign Final Sales	$cov(\phi_H^C, \phi_{FH}^P)$	0.63	0.54
Cov. Firms' Domestic Final Sales and Employment	$cov(\phi_H^C, \phi_H^L)$	0.06	0.11
Cov. Firms' Exports Productivity and Employment	$cov(\phi_{FH}^P, \phi_H^L)$	0.21	0.32
Var. Exports Quality Shifter	$v(\phi_F^C)$	0.62	0.43
Var. Import Prices	$v(\phi_{HF}^P)$	0.62	0.52
Var. Trade Costs	$v(\tau_F)$	0.42	0.32
Cov. Exports Quality Shifter and Import Prices	$cov(\phi_F^C, \phi_{HF}^P)$	0.08	0.12

Notes: Moments related to firms' and foreign primitives distribution's parameteres, generated from the model and the data. Also, the table includes the parameters each moment is intuitively related to in the model.

Table D.6: Model's Fit to Aggregate Moments of Production Costs

Moment	Related Parameter	Model	Data
Aggregate Labor Share	α^L	0.67	0.71
Aggregate Import to Domestic Inputs	α^I	0.61	0.65

Notes: Moments related to aggregate costs of production, generated from the model and the data. Also, the table includes the parameters each moment is intuitively related to in the model.

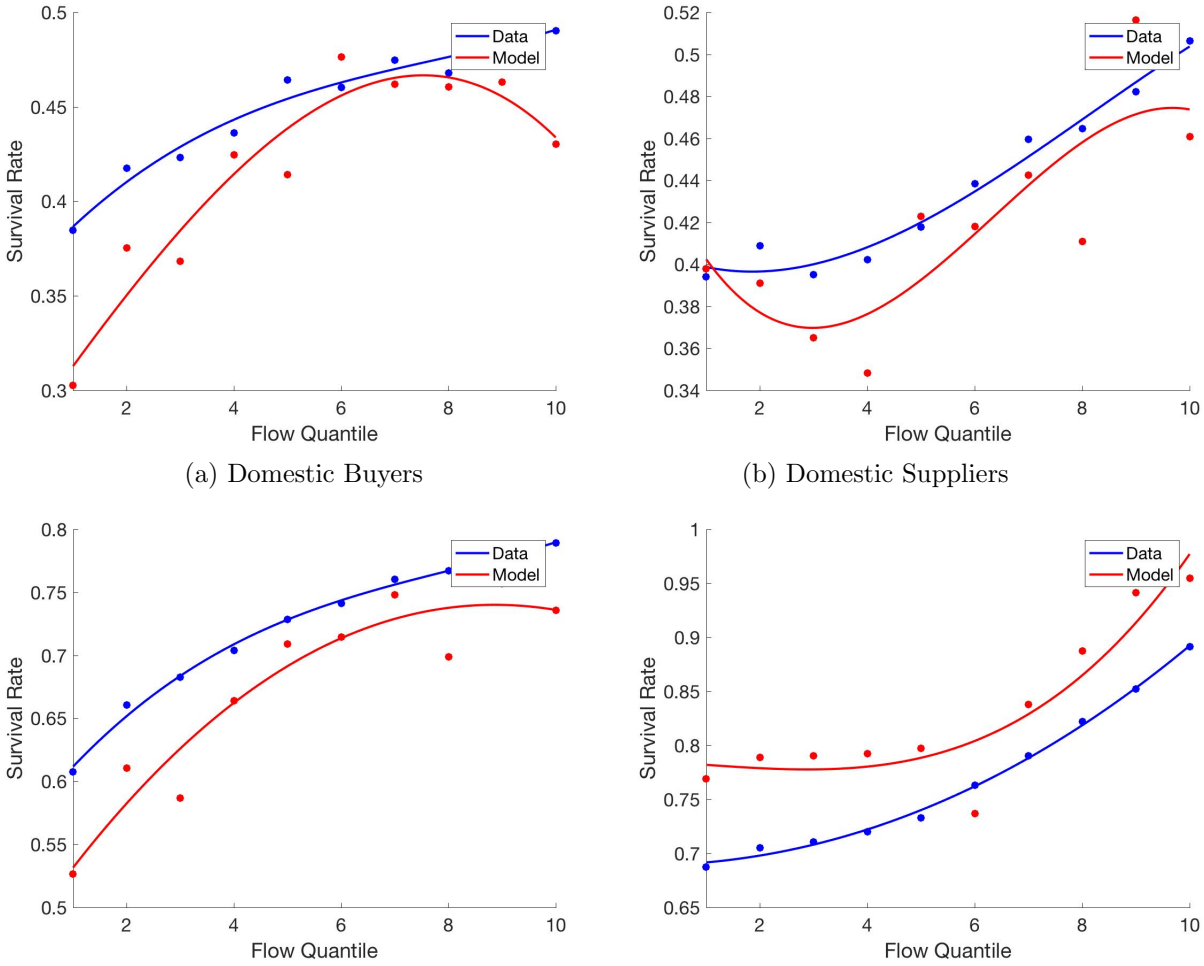
D.3 Results from the SMM

The fit of the model to moments that were targeted in the SMM procedure are presented in Table D.5, D.6, and Figures D.2, D.4. Table D.5 shows the fit of the model to moments related to firm size measures and international trade characteristics and Table D.6 shows the fit to aggregate moments such as the labor share. Figure D.2 shows the relationship between the fraction of links that firms retain between years and measures of firm size. Figure D.4 shows the relationship between the number of links and measures of firm size. Both the tables and figures show that the model is able to replicate those moments fairly well. Table D.7 and D.8 shows the fit of the model to regressions ran in the data. As previously discussed, these regressions are motivated by relationships established in the model. The model is able to replicate the main patterns of these regressions.

D.4 Inference of International Primitives

This appendix describes how to extract primitives from international firms ($\phi_F = \{\phi_F^C, \phi_{HF}^P\}$) and iceberg trade costs τ_F from the available data, conditional on parameters and the structure of the model. In other words, how to invert the model in order to go from the data to the model's international primitives. The strategy to invert the model is the following. I use the optimality

Figure D.2: Survival Rates and Size - Unweighted - 2006



(a) Domestic Buyers

(b) Domestic Suppliers

(c) Foreign Buyers

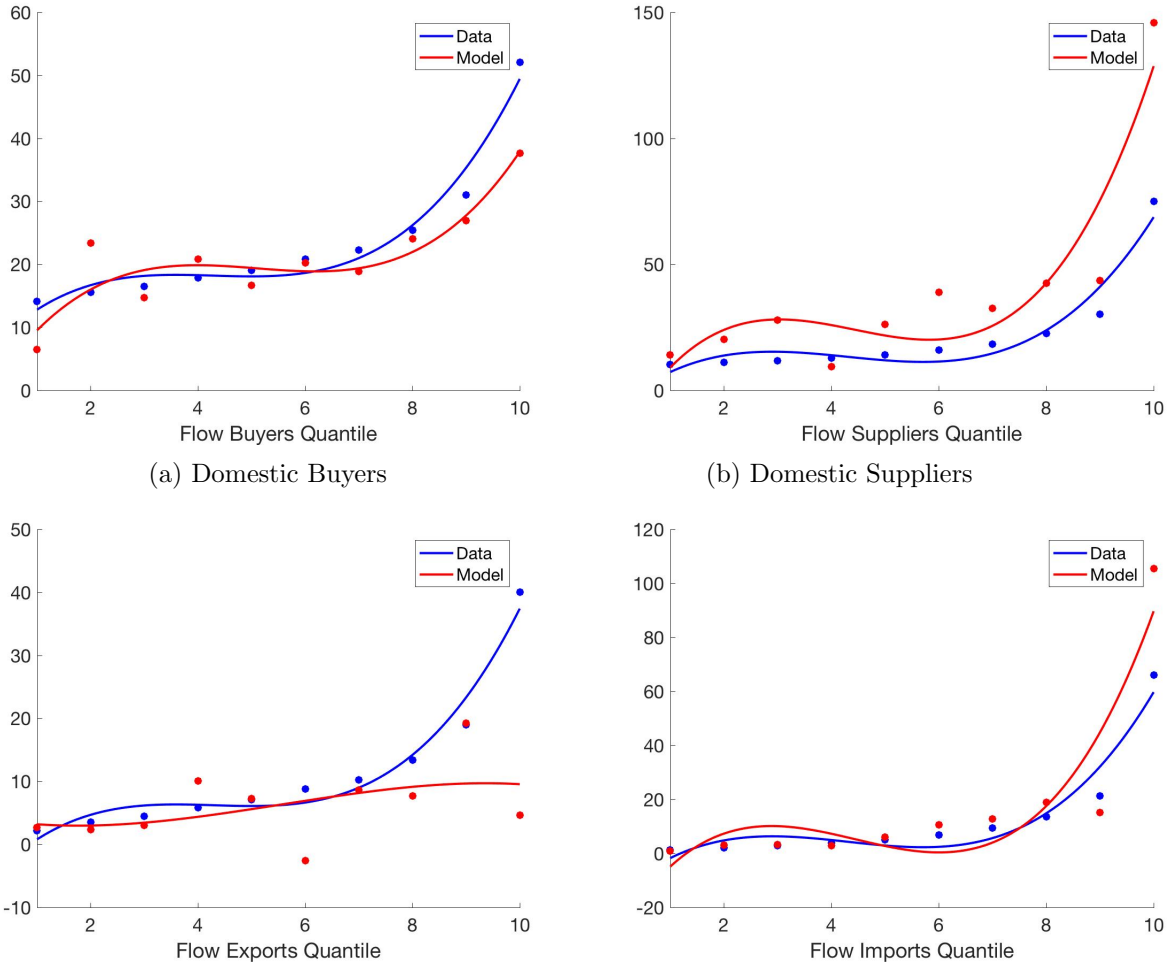
(d) Foreign Suppliers

Notes: These figures document survival rates of intermediate input links, both in domestic and foreign markets, as well as on the buyer and supplier side. These rates are shown using the data and model-simulated data. Survival rates are the fraction of links at a given time that continue the following year for each firm. The measure of size in each graph is the total flows of each firm in the corresponding margin. For example, the size measure of the survival rate of domestic suppliers is the total expenditure in domestic suppliers of each firm. Qualitatively similar graphs can be obtained using different measures of size such as total sales. The problem with this variable is that it is not observed across the whole distribution because a subset of firms export or import from international markets. Links in this evidence are not weighted and they are documented for 2006.

conditions from the export side of the model to back out the quality shifters ϕ_F^C and the trade cost τ_F. With the trade cost at hand, I evaluate the shape of the distribution and use this to simulate the trade costs from import prices, which in turn allows me to extract the Hicks-neutral productivity terms ϕ_{HF}^P.

Optimality on the export side implies the following:

Figure D.4: Number of Links and Size - 2006



(a) Domestic Buyers

(b) Domestic Suppliers

(c) Foreign Buyers

(d) Foreign Suppliers

Notes: These figures show the number of links that firms had in 2006 and measures of firm size. Domestic links are defined at firm-to-firm pairs whereas foreign links are defined at the firm-to-market pairs. Markets are defined as product-country combinations, where products are defined at the 6-digit HS codes level of aggregation. This is the most disaggregated level the data allows me to get. In the model, the domestic number of links are defined as the number of non-zero entries in the (ϕ_H, ϕ'_H) matrix, where ϕ_H is the buyer and ϕ'_H is the seller. Similar logic applies for international links.

$$x_F^P(\phi_F, \phi_H) = \left(\phi_{FH}^P\right)^{\sigma_{FH}^P - 1} \left(p_F(\phi_F, \phi_H)\right)^{-\sigma_{FH}^P} c_F(\phi_F) \left(P_F(\phi_F)\right)^{\sigma_{FH}^P} \tag{60}$$

In the international trade data, we observe flows and quantities which would allow us to back out unit values. Nevertheless, export flows are defined in terms of *free on board* (FOB). Thus, we can only back out $p_F^{FOB}(\phi_F, \phi_H) = p_F(\phi_F, \phi_H)/\tau_F$. Thus, we have the following relationships:

Table D.7: Propagation of International Trade Shocks: Model's Fit

	Δ Log Sales	
	Data	Model
Direct Import Shock	-0.768***	-0.621
	[0.121]	
Upstream Import Shock	-0.830**	-0.753
	[0.415]	
Downstream Import Shock	0.857**	0.931
	[0.429]	
Direct Export Shock	0.318***	0.243
	[0.105]	
Upstream Export Shock	-0.513	-0.954
	[0.719]	
Downstream Export Shock	0.399***	0.543
	[0.084]	
N	13752	15000
R^2	0.971	0.642
Firm FE	✓	
Mean DV	15.07	10.12
SD DV	2.02	1.51

Notes: Results of OLS regressions of Equation (4) ran in the data and in the model. *** p<0.01, ** p<0.05, * p<0.1

Table D.8: International Trade Regressions: Model's Fit

	Δ Log Import Share		Δ Log Export Share		Δ Log Group Export Share	
	Data	Model	Data	Model	Data	Model
Δ Log Import Price	-2.134***	-2.1				
	[0.301]					
Δ Log Labor-to-Int. Ratio	1.432***	1.4				
	[0.531]					
Δ Log Export Price			-3.243***	-3.2		
			[0.801]			
Δ Log Group Export Price					-2.621***	-2.6
					[0.721]	
N	25310	23210	14230	13120	11850	10364
R^2	0.971		0.709		0.742	
Firm and Year FE	✓		✓		✓	

Notes: This table shows the results of the regressions specified in the appendix XXX, implemented in the data and in the model. *** p<0.01, ** p<0.05, * p<0.1

$$\frac{e_F^{c,FOB}(\phi_F,\phi_H)}{\left(p_F^{FOB}(\phi_F,\phi_H)\right)^{1-\sigma_{FH}^P}} = \left(\phi_{FH}^P\right)^{\sigma_{FH}^P-1}(\tau_F)^{-\sigma_{FH}^P} c_F(\phi_F)\left(P_F(\phi_F)\right)^{\sigma_{FH}^P} \quad (61)$$

$$\frac{e_F^{c,FOB}(\phi_F,\phi_H)}{\bar{e}_F^{c,FOB}(\phi_F)}\left(\frac{p_F^{FOB}(\phi_F,\phi_H)}{\bar{p}_F^{FOB}(\phi_F)}\right)^{\sigma_{FH}^P-1} = \left(\frac{\phi_{FH}^P}{\tilde{\phi}_{FH}^P}\right)^{\sigma_{FH}^P-1} \quad (62)$$

where $e_F^{c,FOB}(\phi_F,\phi_H) = p_F^{FOB}(\phi_F,\phi_H)\,x_F^P(\phi_F,\phi_H)$.[73] Plugging back (62) into the definition of the aggregate demand and ideal price index at the product level, one is able to measure $c_F(\phi_F)\left(\tilde{\phi}_{FH}^P\right)^{1-\sigma_{FH}^P}$ and $P_F^{FOB}(\phi_F)\left(\tilde{\phi}_{FH}^P\right)^{\sigma_{FH}^P-1}$, where $P_F^{FOB}(\phi_F) = P_F(\phi_F)/\tau_F$. Note that, I also observe, for each product, the total cost, insurance and freight (CIF) imports from Chile, the home country. Thus, using the following relationship from the model at the product level:

$$\frac{e_F^c(\phi_F)}{c_F(\phi_F)}\left(\tilde{\phi}_{FH}^P\right)^{\sigma_{FH}^P-1} = \tau_F P_F^{FOB}(\phi_F)\left(\tilde{\phi}_{FH}^P\right)^{\sigma_{FH}^P-1} \tag{63}$$

where $e_F^c(\Phi) = P_F(\phi_F)\,c_F(\phi_F)$, one can recover τ_F for exported products. Finally, using the optimality at the product level, one has the following relationships:

$$\frac{e_F^{c,FOB}(\phi_F)}{\left(P_F^{FOB}(\phi_F)\right)^{1-\sigma_F^G}}\left(\frac{1}{\left(\tilde{\phi}_{FH}^p\right)^{\sigma_{FH}^P-1}}\right)^{1-\sigma_F^G} = \left(\left(\tilde{\phi}_{FH}^P\right)^{\sigma_{FH}^P-1}\phi_F^C\right)^{\sigma_F^G-1} D_F \tag{64}$$

$$\frac{e_F^{FOB}(\phi_F)}{\bar{\bar{e}}_F^{FOB}}\left(\frac{P_F^{FOB}(\phi_F)}{\bar{\bar{P}}_F^{FOB}}\right)^{\sigma_F^G-1} = \left(\frac{\phi_F^C}{\dot{\tilde{\phi}}_F^C}\right)^{\sigma_F^G-1} \tag{65}$$

In short, by using firm-product level FOB export flows and prices, product level CIF import flows and elasticity values, one can recover from the model $\left(\frac{\phi_{FH}^P}{\tilde{\phi}_{FH}^P}\right)^{\sigma_{FH}^P-1}$, τ_F and $\left(\frac{\phi_F^C}{\dot{\tilde{\phi}}_F^C}\right)^{\sigma_F^G-1}$.[74]

Given τ_F, one has the following from CIF import prices:

$$p_F^G(\phi_H,\phi_F) = \frac{\tau_F}{\phi_{HF}^P} = p_F^G(\phi_F) \tag{66}$$

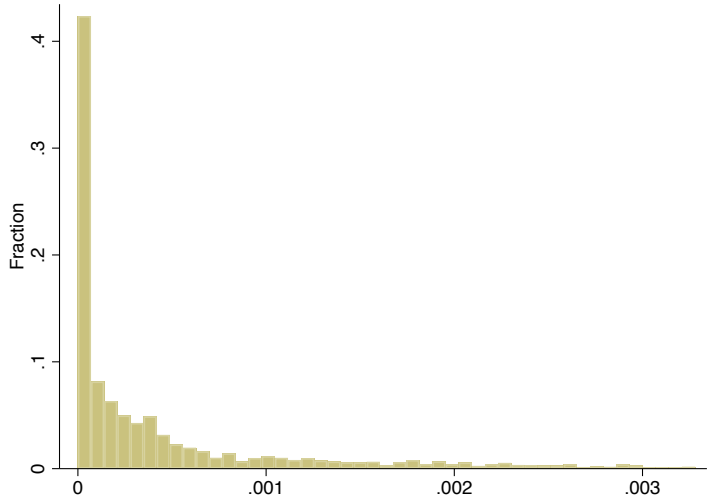
which implies that ϕ_{HF}^P can be recovered almost directly from import CIF prices and trade costs.[75] Given the inversion of the model and exogenous parameter values as an example ($\sigma_{FH}^P = \sigma_F^G = 4$), Figure D.6 shows that distribution of τ_F is Pareto. The estimated distributions of ϕ_F^C and ϕ_{HF}^P are discussed in Section 5.

[73] Variables with a bar are averages across varieties within products: $\bar{x}(\phi_F) = \int x(\phi_F,\phi_H)\,dF_{\phi_H}(\phi_H)$. Variables with a tilde are CES aggregates across varieties: $\tilde{x}_{FH}^P = \left[\int x(\phi_H)^{\sigma_{FH}^P-1}\,dF_{\phi_H}(\phi_H)\right]^{\frac{1}{\sigma_{FH}^P-1}}$.

[74] Variables with double bars are averages across products and varieties: $\bar{\bar{x}} = \int\int x(\phi_F,\phi_H)\,dF_{\phi_F}(\phi_F)\,dF_{\phi_H}(\phi_H)$. Variables with a dot and a tilde are CES aggregators across products $\dot{\tilde{x}}_F^G = \left[\int x(\phi_F)^{\sigma_F^G-1}\,dF_{\phi_F}(\phi_F)\right]^{\frac{1}{\sigma_F^G-1}}$.

[75] This would work perfectly if one had at least one import product, for every export product. In practice, the opposite happens in the data for Chile. Chile exports to approximately 20 thousand product-country pairs each year and imports from approximately 60 thousand each year. In order to overcome this limitation, I infer the distribution of trade costs and simulate from that distribution the trade costs for the imported products from which there are no exports from Chile.

Figure D.6: Iceberg Trade Cost τ_F - 2006



Notes: This figure reports the distribution of τ_F using the derivations of Section D.4, setting $\sigma_{FH}^P = \sigma_F^G = 4$ as an example and using data from 2006.

D.5 Structural Relationships to Estimate the Elasticities of Substitution

This subsection shows the relationship between firms' outcomes and inputs, and the elasticities of substitution. The model predicts highly non-linear relationship between these. Thus, instead of running these exact relationships to identify the elasticities of substitution, I run auxiliary regressions, in the spirit of indirect inference, that relate these variables indirectly. The main relationships are the following:

$$\begin{aligned}
\log\left[\left(\frac{r(\phi)}{tvc(\phi)}+\frac{\sigma_H^P}{1-\sigma_H^P}\right)tvc(\phi)\right] &= \kappa_0+\frac{1}{\sigma^P-1}\log s^L(\phi)+\frac{\sigma_H^C}{\sigma_H^C-1}\log s_H^C(\phi) \\
&\quad -\log(\phi_H^P\phi_H^C) \qquad (67)\\
\log\frac{s_F^X(\phi)}{s^X(\phi)} &= \kappa_1+(1-\sigma^X)\log p_F^X(\phi)+\frac{\sigma^X-1}{\sigma^P-1}\log\left(\frac{s^L(\phi)}{s^X(\phi)}\right) \\
&\quad +(\sigma^P-1)\log\phi^L \qquad (68)\\
\log s_F^C(\phi) &= (1-\sigma_F^C)\log p_F^C(\phi)+\sigma_F^C\log\phi_F^C \qquad (69)
\end{aligned}$$

where κ_0, κ_1 are constants, $tvc(\phi)$ and $r(\phi)$ are firm ϕ's total variable costs and revenues, respectively. $s^L(\phi)$, $s_F^X(\phi)$ and $s^X(\phi)$ is the share of total variable costs going to ϕ's wage bill, foreign intermediate inputs (imports) and overall intermediate inputs expenditures, respectively. $s_H^C(\phi)$ and $s_F^C(\phi)$ is ϕ's share of domestic and foreign final demand relative to total domestic and final demand, respectively. Each of these relationships highlight how changes to inputs affects firms' outcomes.

The first relationship is firms' revenue functions for firms that do not export.[76] It shows that

[76] Export status introduces even more non-linearities in the revenue function. I omit this here to gain intuition.

changes to firms' labor share and final demand affect firms revenues depending on the elasticities of substitution both in demand and production. Since shocks to firms' imports could affect the labor share, the marginal costs and thus also final demand, the relationship between firms' sales and those shocks in the propagation regressions from Equation (4) are mediated by the elasticities of substitution. This logic holds for both the direct and indirect shocks received by the firm.

The second relationship comes directly from the sourcing strategy of firms. It shows how firms' sourcing is a function of the price that firms face when importing and the labor share they have. Finally, the third relationship is a standard CES demand system for exports. Note that these two regressions also have the challenge that there is an underlying selection stage into importing and exporting that introduces non-linearities to them.

Finally, another dimension that is challenging in running these directly these regressions, besides the non-linearities is the fact that in models with fixed costs the relevant measure to compute factor shares is total variable costs, but in standard firm data one can only observe total costs. Thus, all the shares used in those regressions cannot be appropriately computed with standard data.

Appendix E Counterfactuals

E.1 Decomposition of International Trade Shocks

This appendix shows that international shocks are not driven separately by country nor product variation. Rather, it shows that the majority of the variation comes within countries, across products. There is also significant variation within products, across countries, but less than the former. The purpose of doing this is to evaluate whether potentially these shocks capture variation such as exchange rate movements, which vary at the country level. Since all measures of shocks are in US dollars, they take into account variation in exchange rates. Exchange rates variation is not the best for the analysis in this paper since it could capture endogenous variation of the domestic market's environment, thus challenging the exogeneity of the shocks.

To show how relevant this concern might be, I take different measures of international shocks at the market-year level,[77] and evaluate how much country-year fixed effects and product-year fixed effects account for its variation. To do so, I implement the following regressions:

$$\Delta \log y_{cpt} = \gamma^{C}_{ct} + \epsilon_{cpt} \tag{70}$$

$$\Delta \log y_{cpt} = \gamma^{P}_{pt} + \epsilon_{cpt} \tag{71}$$

Both equations regress shocks $\Delta \log y_{cpt}$ which are the first difference of variable y of markets defined by country c and 6-digit HS product code p, at year t, on a set of fixed effects. Equation 70, projects the shock to y on country-year fixed effects, whereas Equation 71 does it on product-year

The same arguments exposed here hold when conditioning on exporters. The additional complication is that when conditioning on exporters, one needs to take into account the non-linear selection equation.

[77] Recall that markets are defined as country-product combinations, where products are 6-digit HS codes.

Table E.9: Variance Decomposition of International Shocks (%)

Shock	Margin	Countries	Products
Imports	Flows	7.1	14.0
	Prices	1.1	4.1
Exports	Flows	7.9	20.0
	Prices	1.4	6.3
	Quality	8.9	14.0

Notes: This table presents evidence of a variance decomposition of international shocks. Each entry represents the adjusted R^2 (multiplied by 100) of a regression performed at the country-product-year level, of the shock stated in the columns on year fixed effects interacted with the fixed effect of different dimensions, namely, country or 6-digit HS product codes.

fixed effects. The purpose is to evaluate how much do these fixed effects capture the variation of the dependent variable, represented by the adjusted R^2 of these regressions. The regressions are implemented by ordinary least squares on the following shocks: import flows, import prices, export flows, export prices and the export quality shock described theoretically in Appendix D.4. All these shocks are described quantitatively in Appendix **??**.

The results of the analysis are presented in Table E.9. It presents the adjusted R^2 of the regressions of Equations 70 and 71, multiplied by 100. Thus, each represents what percentage of the variation of the respective shock, do country or product variation account for. The evidence in the table shows that country variation represents not more than 8.9 percent of the variation of international shock, in the case of export quality shocks, and can represent as little as 1.1 percent in the case of import price shocks. Similarly, product-year variation does not account for the majority of the variation of the shocks. Although higher than country-year variation, it never exceeds 20 percent (the case of export flow shocks) and it gets as little as 6.3 percent with export price shocks. This evidence suggests that the main source of variation of these shocks comes from within countries, across products and also within products, across countries. This downgrades the concern of much of the shocks are driven by exchange rate variation and thus gives more support to the exogeneity of the shocks.

Appendix F Data

F.1 General Description of the Data

This appendix describes each of the four datasets used throughout the paper.

Firm-to-Firm Transaction Dataset This dataset corresponds to annual firm-to-firm domestic transactions for the 2003-2011 period. In 2011, there were around 17 million firm-to-firm transactions. This information is provided through a tax form that firms have to fill under the purpose of monitoring VAT payments. In this form firms have to report the complete list of suppliers and buyers that they have in a given year. For each relationship that they report, they have

to include the corresponding amount of the VAT payment involved. The VAT is a flat rate of 19 percent in Chile. There are no domestic exemptions.[78] Given this, one can back out the value of the transaction by the following simple rule:

$$Net\ Value = \frac{Tax\ Payment}{0.19}$$

where *net value* corresponds to the value of interest, i.e., the amount of the transaction net of the tax payment, since this tax is paid by final consumers and therefore returned to producers. Only firms that have total expenditures on intermediates in a given year above $US\$390,000$ have to report this information. This threshold has to be crossed only once. That is, if a firm has expenditure in a given year above the threshold and the next year it is below, then it still has to fill the tax form. Section F.4 discusses the potential biases generated by this cutoff. Overall, these concerns are downgraded by the fact that the firms that report the information, account for around 80 percent of value added in the economy. Finally, Section F.5 presents basic descriptive statistics of the dataset. The structure of this dataset is similar to the ones used recently for Belgium (Magerman et al., 2015)[79] and Japan (Carvalho et al., 2016).[80]

Other Firm Characteristics Dataset This second dataset involves standard annual firm level characteristics for the 2004-2015 period. It includes three characteristics that are important for the analysis in this paper. The first is total sales. This variable corresponds to the sum of firm-to-firm sales and sales to final consumers. This allows to identify what share of firms' total sales corresponds to sales of intermediates. The second is the main 6-digit industry. The SII uses an adapted version of the *International Standard Industrial Classification of All Economic Activities* (ISIC) of the United Nations (UN). Although the mapping with the Standard Industrial Classification (SIC) or the North American Industry Classification System (NAICS) used in the US is possible, it comes at the cost of losing two levels of aggregation. That is, the concordance between the SII's codes and NAICS can be done but at the 4-digit industry level. Unless otherwise noted and in order not to lose information, we will refer to an industry as the SII classification.

Chile's International Trade Dataset This dataset provides detailed information of firms' international trade activity. It is collected by the Customs public agency for the 2004-2015 period. It reports by law for each firm the flow value and quantity of international goods transactions (whether it is an export or import) detailed at the product and country level. That is, it reports the characteristic of transactions that each firm has with every country and every product they export or import. The product definition is at the 6-digit level from the Harmonized System (HS) classification.

Global International Trade Dataset The domestic administrative datasets just described

[78] Exports do not have to pay this tax.

[79] Belgium's data has a different cutoff. There all firms have to report their production networks, but there is a reporting cutoff on firm pairs. This means that conditional on reporting production links, some links will not be reported.

[80] Although the one from Japan comes not from an administrative source, but from a private credit reporting agency.

are complemented with global trade flows at the product-country level for the 2002-2014 period. The raw source of this dataset is a repository of official international trade information collected by the UN Statistical Division which is called COMTRADE. This dataset is organized and cleaned by the *Centre d'Etudes Prospectives et d'Informations Internationales* (CEPII, for its acronym in french). CEPII is a French private research center in international economics which is part of the network coordinated by the Economic Policy Planning for the Prime Minister. CEPII organice COMTRADE's database into BACI, which is a cleaned version of COMTRADE's database. This dataset, identifies the value and quantity for each 6-digit HS product and country that is traded globally. This dataset is key for building the international trade shock which provides the source of identification of the reduce form exercise and estimation.[81]

Merging the Chilean datasets Each of the domestic administrative datasets is merged using the unique and time-invariant tax ID that each firm has. Whenever a firm starts reporting taxes, they obtain this tax ID that they use for all administrative processes.[82] Throughout the paper, a firm will be defined as a tax ID that has positive sales, more than 5 full-time equivalent workers (FTE) and positive intermediate input expenditures.[83],[84]

F.2 Firm Definition

The definition of a firm in these datasets is a tax ID. As in other administrative datasets that use tax IDs to identify firms (Song et al., 2018). This gives rise to two potential problems in terms of identifying a firm. The first is that a multi-plant firm can have several tax IDs. The second is that a tax ID can be used by citizens to avoid taxes by charging personal expenditures as part of expenditures of a fake firm. In order to avoid this problem, tax IDs with less than 5 FTE workers.[85] will be excluded from the analysis. Given these two problems, I follow a definition which is close to the economic definition of a firm that the SII uses.[86] A firm is defined as a tax ID that, in a given year, has:

1. Positive total sales.

2. More than 5 FTE workers.

[81]The merge between this global dataset and the international trade dataset at the firm level from the Customs agency, is described in the Appendix F.9.

[82]In order to keep the confidentiality of the datasets, I have access to the datasets through the SII with a fake ID and I don't have access to the crosswalk between this ID and the real tax ID.

[83]The details for justifying these cutoffs are explained in the Appendix F.2.

[84]Since it is the first time that these datasets are merged in Chile, the process of merging them is described in detail in the Appendix F.3.

[85]Since the employer-employee data has no information on hours, I define a full-time equivalent worker as the share of a worker's total labor income coming from a particular job.

[86]In fact, this is an extended version of the SII firm definition. The SII qualifies a tax payer as a firm if it meets at least one of the following requirements:

- Submits the DJ1887 form, which contains information on the firms employees under contract and its wages.
- Submits the DJ1879 form, which contains information on the firms employees without contract and its wages.
- Is a VAT taxpayer.

Table F.10: Number of Tax IDs and Firms

Year	Sales	Employment	Materials	Exports	Imports	Total	Firms
2005	863,113	472,901	1,289,872	6,592	35,969	1,713,125	277,384
2006	883,192	478,196	1,280,356	6,662	39,696	1,709,243	276,022
2007	895,042	484,887	1,272,574	7,608	42,456	1,703,079	277,121
2008	907,071	491,340	1,264,107	7,900	70,950	1,724,194	274,852
2009	919,798	494,988	1,257,309	7,179	75,025	1,730,464	273,850
2010	936,802	502,500	1,253,387	7,116	86,171	1,741,914	277,441
2011	962,646	519,059	1,244,638	7,348	96,036	1,753,329	285,748
2012	988,743	522,943	1,240,262	7,205	114,865	1,776,272	287,902
2013	1,014,482	545,788	1,229,674	7,336	138,919	1,802,013	299,525
2014	1,045,046	553,187	1,513,992	8,019	149,834	1,922,023	300,243

Notes: This table shows the number of tax IDs that have non-missing values for each variable in the columns and for each year.

3. Positive expenditure on intermediate inputs.

The first and second condition is to ensure that one works with tax IDs that are involved in a production process in a given year. The final condition is to consider tax IDs that belong to the production network of the economy, which are the ones relevant for this paper.

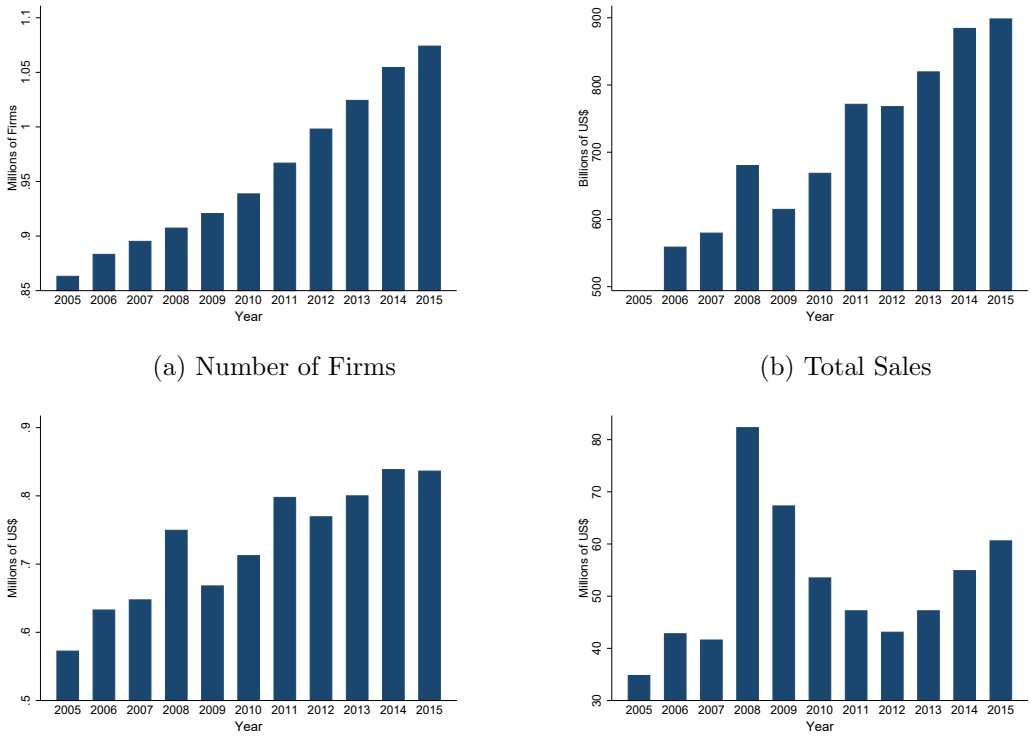
Table F.10 presents the details of the number of tax IDs available in each of the datasets. It shows that the sales and materials variables have a significantly large number of tax IDs reporting that information. When all these datasets are pooled together, the number of tax IDs becomes around 1.7 millions. When the aforementioned conditions are applied, the number of tax IDs decreases to around approximately 300,000.

F.3 Database Merge

This section describes the merge between the different datasets used from the SII. I first section describe the baseline dataset which includes yearly firm level sales estimates. Then I describe the list of firm level datasets which are merged to sales data. Finally I provide descriptive statistics on the share of baseline firms that were properly matched with each dataset.

Baseline Sales Dataset The SII provides a dataset with their internal estimation of yearly firm level sales, between year 2005 and 2015. The methods by which this dataset is constructed are protected and confidential, and thus not available to any external (non-SII) research team. This estimations are used by SII in their tax collection procedures and audits. Figure F.7 provides descriptive statistics for the sales dataset. Figure F.7a shows, the number of firms increases steadily from close to 0.85 millions in 2004 to 1.1 millions in 2015. Figure F.7b shows the total sales level in the economy, which starts at around 550 billions in 2004, rises to near 700 billions in 2008, and decreases in 2009 to 600 billions. Only in 2011 the economy exceeds the pre financial crisis overall sales levels, at near 800 billions, from were it grows to around 900 billions in 2015. Figure F.7c shows the average firm sales level, which mirrors the distribution of the overall sales level. The

Figure F.7: Firm Sales Distribution



(a) Number of Firms

(b) Total Sales

(c) Average Firm Sales

(d) Sales Standard Deviation

Notes: This figures present descriptive statistics for sales of firms available in the SII dataset. Values are in 2014 US$ dollars. Note that none of these statistics were subject to any censoring or windsorization treatment, and thus correspond to that of the original information provided by SII.

average firm sales grows from near 600 thousand dollars in 2005, to more than 800 thousand dollars in 2015. Finally, Figure F.7d shows the standard deviation of firm level sales. It starts at around 30 millions in 2004, peaks at 80 millions in 2008, and reaches 60 millions by 2015. These values show how the dispersion of firm size increased during the Great Recession.

There are five datasets that are merged with the sales dataset: intermediate input expenditures, firm-to-firm transactions, international trade and firms' location. I describe each of them in turn and then how the merge works with the sales dataset.

Materials (F29) SII's F29 form collects firm level monthly VAT[87] tax credits and debits, this data is available from 2005 to 2015. The reporting firm must present its unique Taxpayer Identification Number (Rol Unico Tributario - henceforth RUT), as well as the monthly sum of tax credit and tax paid. Thus, by multiplying the values by the inverse of the VAT tax rate, we obtain the yearly sum of sales and purchases. A subset of the codes collected at the F29 form allow us to

[87]The Chilean Impuesto al Valor Agregado - IVA (Value Added Tax - VAT) has a flat rate of 19 percent. Declarations and payment of the VAT must be made monthly and its amount is calculated based on the difference between the tax paid (by selling) and tax credit (by purchasing). If the latter is higher, the difference can be used as tax credit for later payments.

identify or estimate the level of investment, imports and material purchases at the firm level.

Firm-to-Firm Networks (F3323) SII's F3323 form contains data on firm-to-firm purchases and sales. This data is available from 2003 to 2011, when the F3323 was replaced by forms F3327 and F3328. The reporting firm must present its unique RUT and that of its partner-firm, as well as the yearly sum of tax credit and tax paid from transactions between the pair of firms, for all partner-firms that the reporting firm has. Thus, by multiplying the values by the inverse of the VAT tax rate, we obtain the yearly sum of sales and purchases between both firms. Besides sales and purchases, the F3323 form includes credit and debit notes issued or received by the declaring firm. Credit and debit notes enforce certain tax exemptions or benefits. We restrict this dataset to the universe of firms declaring the F3323 form, thus not including firms declared at the form. This firms tend to be larger than average, as the submission of the F3323 is only required for firms subject to the VAT tax and exporters[88] whose past year annual sum of credits is higher than $250.000.000 Chilean pesos, approximately equivalent to US$385 thousand. More details about this dataset can be found in the Appendix F.4.

Firm Customs Exports and Imports (Exports and Imports) The Chilean Customs Authorities (*Servicio Nacional de Aduanas*), through the Secretary of Treasury, provided imports and exports transactions data between the years 2002 and 2015. The dataset contains information on the transaction values (free on board (FOB) for exports and cost, insurance and freight (CIF) for imports), the quantity of goods associated to the transaction, the product code according to the 6 Digits Harmonized Commodity Description and Coding System, and the country of destination of exports and country of origin of imports.[89] Further details about this dataset can be found in the Appendix F.9.

Firm Headquarters Location (Location) SII provided data on each firm headquarters location. The location corresponds to the comuna, the Chilean smallest administrative subdivision. Chile's 346 communes are grouped into 54 provinces, which are themselves grouped into 15 regions (13 before 2007).[90]

Merge Quality The quality of the merge between these datasets and the one of firm's sales is described in Table F.11. Around 80 percent of the firms in the sales dataset are matched with the materials dataset. This high share is explained because the F29 form has no submission requirements but being a VAT taxpayer, which is a weak requirement in Chile. Around 3 percent of firms in 2011 from the firm-to-firm dataset are merged with the sales dataset. Even tough the share of firms that report their production network is small, they account for a large fraction of the economy, as they are usually large.[91] The share of firms that import and export is around 4 and 1 percent, respectively. Finally, in terms of location, most firms are merged with the sales dataset (99.7 percent).

[88] Exporters are exempted from the VAT on their foreign sales and have a right to a reimbursement for the VAT paid as part of their exporting activities.

[89] The countries are classified according to the Chilean Customs Classification System, available in this link.

[90] The official list of comunas is available in this link.

[91] For more details about this and other concerns about the coverage of the firm-to-firm dataset, refer to the Appendix F.6.

Table F.11: Matched Shares Relative to Firms' Sales Dataset

Year	(1) Materials	(2) Firm-to-Firm	(3) Imports	(4) Exports	(5) Location
2005	80.09	00.76	02.81	00.72	99.62
2006	80.08	00.97	02.98	00.71	99.77
2007	80.85	01.14	03.11	00.77	99.80
2008	80.99	01.54	03.87	00.79	99.82
2009	80.74	02.02	03.83	00.73	99.68
2010	80.82	02.52	04.02	00.71	99.66
2011	81.21	02.77	04.22	00.71	99.61
2012	81.14	.	04.38	00.68	99.67
2013	81.61	.	04.59	00.67	99.70
2014	82.03	.	04.53	00.69	99.78
2015	82.76	.	.	.	99.93

Notes: This table shows the percentage of firms in the baseline sales dataset that are matched with different datasets, using the unique tax ID (RUT). Dots indicate that the dataset is not available that year.

F.4 Production Networks Dataset Description and Cleaning

F3323 Form The purpose of this data appendix is to describe the main source of firm-to-firm links that constitute the basics to study domestic production networks at the firm level. The data used comes from Chile. To the best of my knowledge, this chilean dataset has not been used for research purpose. Thus, the goal of the appendix is to be transparent in describing the dataset, how it was cleaned and some basic statistics. The administrative source of firm-to-firm links is the F3323 form that firms have to report to the SII (the chilean equivalent of the US' IRS). This form summarizes annual value added tax (VAT) declarations arising from firm-to-firm trade. Firms subject to the VAT tax[92] and exporters[93] whose past year annual sum of credits is higher than $250.000.000 Chilean pesos, approximately equivalent to US$385 thousand, are required to submit the F3323 form once a year (in April). This form was introduced in 2003 by the Chilean Internal Revenue Services (Servicio de Impuestos Internos - henceforth SII) and replaced in 2011 by more complete forms.[94] This implies that our firm networks dataset spans from 2003 to 2011.

F3323 Content Each F3323 form contains data on firm-to-firm purchases and sales. The reporting firm must present its unique Taxpayer Identification Number (Rol Unico Tributario - henceforth RUT) and that of its partner-firm, as well as the yearly sum of tax credit and tax paid from transactions between the pair of firms, for all partner-firms that the reporting firm has. Thus,

[92]The Chilean Impuesto al Valor Agregado - IVA (Value Added Tax - VAT) has a flat rate of 19 percent. Declarations and payment of the VAT must be made monthly and its amount is calculated based on the difference between the tax paid (by selling) and tax credit (by purchasing). If the latter is higher, the difference can be used as tax credit for later payments.

[93]Exporters are exempted from the VAT on their foreign sales and have a right to a reimbursement for the VAT paid as part of their exporting activities.

[94]Actually, in 2011 it is replaced by the DJ3327 and DJ3328 forms, which register firm-to-firm purchases and sales respectively, and whose submission is monthly.

by multiplying the values by the inverse of the VAT tax rate, we obtain the yearly sum of sales and purchases between both firms. Besides sales and purchases, the F3323 form includes credit and debit notes issued or received by the declaring firm. Credit and debit notes enforce certain tax exemptions or benefits.

The information contained in the F3323 form consolidates that of all of the firm operational units, which implies that multi-plant firms submit only one form per partner firm. It is worth noting that the data we use comes from the raw firm declarations to SII, and therefore, it does not include any revision or correction by the tax authorities. Furthermore, as aforementioned, it is the first time that this dataset is used in Chile for research purposes. Given this, I explain in detail in the next section how we cleaned the dataset before doing the main analysis of the paper.

F3323 Form Cleaning Methodology To clean the production networks data, we take three steps. First, we define and discuss our treatment of double sided, miscoded or missing transactions, followed by our adjustment to the yearly sum of purchases and sales by the issuing of credit and debit notes. Our second step is to add firm-level information from other sources to asses the likelihood of the purchases and sales values we observe, which we use to perform further cleaning processes. We also present a battery of tests used to detect and correct outliers or time inconsistencies in the data. Finally, we discuss our criteria for defining what is a firm and use this to exclude transactions that might not be economically relevant because they involve non-firm tax units.

Duplicates and Miscoded Transactions The first step is to reduce the dataset to yearly firm-to-firm declarations. Each declaring firm should submit one yearly F3323 form per partner firm, however, there are some duplicates. Most of duplicates arise because of issuing or receiving credit notes. This is because credit and debit notes are used to enforce corrections to the tax credit or tax paid by firms, and are regularly issued some time after the original transaction. In case of duplicate firm pairs in a given year, we calculate the yearly sum of tax credit, tax paid, and credit notes by that firm pair, and use this information as the firm pair observation. Finally, a number of F3323 forms have the same RUT as reporting and partner firm. We attribute this to miscoding and delete these observations.

Double Sided Transactions Next, we deal with double sided reporting. Because the F3323 form is bilateral (contains information on two firms) a firm can be both reporter and reported for the same transaction (if both firms meet the F3323 submission requirement). We transform the dataset into an edge list where each row contains the RUT of the seller, the RUT of the buyer and the value of the transaction, irrespective of which firm (seller or buyer) submitted the F3323 form. Double sided transactions occur if both firms meet the F3323 submission requirements. When a firm that should submit the F3323 is reported by another firm and does not report that specific transaction, we posit that these cases correspond to missing double sided transactions (i.e. there is data for only one side). Our edge list is adjusted to impute this missing observations, thus expanding the dataset.

Credit Notes Correction We use the issuing of credit notes to correct the value of sales and purchases between firms. If the reporting firm issues a credit note, its value is subtracted from

Table F.12: F3323 Statistics.

Year	(1) Original Observations (Millions)	(2) Duplicates (%)	(3) Identical ID's (%)	(4) Two-Sided Observations (%)	(5) Imputed Observations (%)	(6) Positive Credit Notes (%)	(7) Final Observations (Millions)
2003	13.73	00.84	00.01	07.27	00.96	.	13.20
2004	14.15	00.87	00.01	07.76	01.20	.	13.44
2005	14.90	00.92	00.01	09.20	01.49	16.65	13.81
2006	15.14	01.06	00.02	10.45	02.32	21.66	13.55
2007	16.45	00.97	00.02	10.14	02.35	25.90	14.05
2008	18.22	01.60	00.02	11.38	02.34	28.64	15. 10
2009	18.72	02.48	00.02	12.84	02.87	26.66	15.71
2010	21.21	01.28	00.02	13.60	03.21	30.29	17.30
2011	19.61	01.35	00.02	13.40	05.56	31.52	15.73

Notes: Percentages in columns (2) to (6) w.r.t Original Transactions. Credit Notes were not part of the F3323 form until 2005.

the yearly sum of sales of the declaring firm to the declared firm. If the declared firm issues a credit note, its value is subtracted from the yearly sum of purchases of the declaring firm from the declared firm. As credit notes can deduct the value of tax credit or tax debit from transactions between two firms, and because credit notes are not necessarily issued in the same period of the F3323 form submission, the adjustments arising from credit notes can create negative values for the yearly sum of purchases or sales between two firms. If both values (purchases and sales) are found to be negative, one negative and the other zero, both zero or both missing, then the observation is deleted from the dataset. If there is a positive and a negative value, the negative value is imputed with a zero.

The effects of these adjustments to the data are summarized in Table F.12. One can see that the effects on the number of transactions is small except for the imputation of double sided transactions. The final dataset has around 15 million observations per year.

Outlier Transactions We add firm-level information on yearly sales and full time equivalent (FTE) employees to each firm-to-firm interaction to check the presence of outlier transactions. Sales data is based on SII estimates and employment data comes from the DJ1879 and DJ1887 forms.[95] We can then compare the size of individual transactions with respect to the firms' overall sales level, as estimated by the SII. Each selling transaction should not be larger than the yearly sales of the declaring firm, and consequently, each purchasing transaction should not be larger than the yearly sales of the declared firm. If a transaction exceeds this cutoff (the firms' SII estimated sales), its value is replaced with the firm-pair highest value (across all years) that doesn't exceeds the threshold defined by the SII total sales estimate.

We also perform checks for transactions that seem unlikely, such as exceptionally large trans-

[95] Our treatment of the DJ1887 and DJ1879 forms is discussed in the Income Data Appendix.

Table F.13: F3323 Tests.

Year	(1) Total Observations (Millions)	(2) Outlier Purchases (%)	(3) Outlier Sales (%)	(4) Purchases exceeding Total Sales of Seller (%)	(5) Sales exceeding Total Sales of Seller (%)	(6) Time Inconsistent Purchases (%)	(7) Time Inconsistent Sales (%)
2003	13.20	00.00	00.01	.	.	00.01	00.12
2004	13.44	00.00	00.00	.	.	00.08	00.01
2005	13.81	00.00	00.01	00.03	24.45	00.06	00.08
2006	13.55	00.00	00.03	00.03	19.28	00.04	00.05
2007	14.05	00.00	02.03	02.03	19.78	00.03	00.04
2008	15.10	00.00	00.07	00.06	17.94	00.04	00.00
2009	15.71	00.00	00.13	00.07	17.72	00.06	00.05
2010	17.30	00.00	00.21	00.08	17.69	00.00	00.05
2011	15.73	00.00	00.31	00.08	16.21	00.10	00.12

Notes: Columns (6) and (7) are missing for years 2003 and 2004 as SII sales estimates are available from 2005 onwards.

actions at small firms.[96] A firm is considered large if its FTE employees are higher or equal to 200.[97] A transaction is deemed large if it exceeds US$100,000,000. We take the natural logarithm of all transactions and standardize them across reporter firms (for all years). If the absolute value of the log-standardized transaction exceeds a critical value of 2.58 (critical z-value for a two-sided t-test with 99 percent confidence), the firm has less that 200 full time equivalent employees and the transaction exceeds US$100.000.000, then the transaction is tagged as an outlier. Outliers are replaced with the highest value among transactions by the same firm that are not tagged as outliers.

Time Consistency of Transactions Finally, to check the consistency of individual firm-to-firm relations over time we perform the following test. We take the natural logarithm of the value of all transactions and standardize them for a given firm pair. If the absolute value of the log-standardized transaction exceeds a critical value of 2.58 (critical z-value for a two-sided t-test with 99 percent confidence), then the transaction is tagged as time inconsistent. For transactions tagged as time inconsistent, if the standardized value is positive the transaction is deemed upward time inconsistent. We then assume that two firms have a stable relation if they have traded for 5 periods or more. The stability definition is transaction specific, this implies that firms can have a stable purchasing relation, a stable selling relation, or both. If a transaction is tagged as upward time-inconsistent and the firm-to-firm relation is deemed stable, then the time inconsistent value is replaced with the highest value among transactions between both firms from time consistent periods.

The number of transactions for which our methods made corrections are summarized in Table F.13.

[96] The criteria used in these tests follow the work done in cleaning firm-to-firm transactions from Belgium (Magerman et al., 2015).

[97] This is the official threshold for being a large firm in terms of employees.

Table F.14: Firm Definition.

Year	(1) Total Observations (Millions)	(2) Main Firms (1000's)	(3) Real Main Firms (%)	(4) Main Firm Transactions by Real Firms (%)	(5) Partner Firms (1000's)	(6) Real Partner Firms (%)	(7) Partner Firm Transactions by Real Firms (%)	(8) Transactions between Real Firms (%)
2003	.	.	.	.	.	.	.	.
2004	.	.	.	.	.	.	.	.
2005	13.81	6.68	58.60	71.98	882.17	30.40	46.50	40.79
2006	13.55	8.94	57.80	74.95	873.46	30.54	49.64	44.43
2007	14.05	11.02	54.65	72.92	958.61	28.06	48.59	43.22
2008	15.10	16.10	50.16	72.68	939.63	28.37	49.00	39.24
2009	15.71	22.27	49.23	78.36	961.47	27.67	49.79	45.52
2010	17.30	29.36	47.20	75.96	1001.86	26.97	49.91	41.45
2011	15.73	34.59	47.06	74.29	993.24	27.73	51.28	40.59

Notes: Rows for years 2003 and 2004 are missing as SII sales estimates are available from 2005 onwards.

Firm Definition As mentioned in the Appendix F.2, a substantial portion of the dataset is composed of firms that besides from appearing at the F3323 form, have characteristics that are not consistent with a standard firm definition. Following the same definition than in the Appendix F.2, the number of firms and transactions that comply with this firm definition are summarized in Table F.14. We refer to F3323 reporting firms as *Main Firms*, and to firms declared by Main Firms as *Partner Firms*.

As the F3323 is declared by exporters and high-selling firms, the likelihood of fitting our firm definition is higher for main firms than for partner firms, whose inclusion in the data depends only on having traded with the main firm. The number of Main Firms rises from 6.68 thousands in 2005 to 34.59 in 2011, but the share of them that meets the real firm criteria decreases from 58.6 percent in 2005 to 47.06 percent in 2011. The share of transactions accounted by real main firms is around 75 percent, implying that despite their low numbers, real main firms cover much of the economic activity in the data. The number of Partner Firms is much higher than that of Main firms, reaching a million in 2010, however, the share of real firms among partner firms is lower, decreasing from 30 percent in 2005 to 27.73 percent in 2011. The share of transactions were a real partner firm is present is around 50 percent.

Our final cleaning step is to restrict the data to the set of transactions between real firms (both Main and Partner firms meet the criteria for being a real firm), which represent around 40 percent of all transactions. This decision is done for conservativeness, since one cannot be sure of the nature of a transaction with a tax ID that does not meet the minimum requirements for being a firm.

Table F.15: Firm to Firm Trade.

Year	(1) Total Observations (Millions)	(2) Average Transaction Value (Millions of US$)	(3) SD of Transaction Value (Millions of US$)	(4) Total Value of Transactions (Billions of US$)
2005	5.67	0.33	15.72	1,860
2006	6.11	0.55	19.39	3,385
2007	6.29	0.84	22.54	5,263
2008	6.83	1.07	24.76	7,298
2009	7.02	1.27	26.02	8,899
2010	7.39	1.36	27.27	10,024
2011	6.58	1.58	27.64	10,399

Notes: All values are in constant 2014 US$.

F.5 Basic Descriptive Stats of Production Networks Dataset

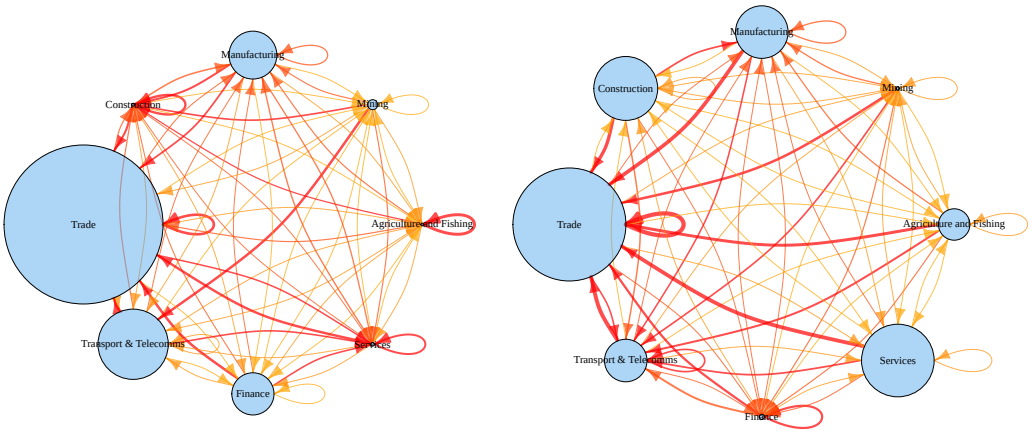
This appendix provides simple descriptive statistics for the cleaned firm-to-firm trade dataset, to which I added firm-level information on industry and geographic location.[98] The dataset spans from 2005 to 2011, which are the periods for which we could apply our firm definition criteria and also the production network data was available.

Table F.15 presents yearly descriptive statistics for the real firms edge list dataset. The dataset has around 7 million links per year. The average transaction value rises from US$0.3 to US$1.4 millions between 2005 and 2011. However, the distribution of transaction values is highly dispersed for all years, as the standard deviation rises from US$16 to US$27 millions in the same time span. The total value of transactions rises steadily from US$ 1,860 billions in 2005 to US$ 10,399 billions in 2011.

Network graphs in Figure F.8 summarize trade flows between industries. It presents the graph of firms' production network aggregated at the industry level. Node sizes are proportional to the industry share of purchases or sales with respect to the economy. Edge width is proportional to the industry pair flow relative to all other flows from the industry of origin. Trade is the largest sector, both in the level of purchases and sales, and it is also the leading trading partner for most industries. Besides firms from the trade sector, firms from the manufacturing, transport & telecomms and finance industries account for most of purchases. The distribution of sales across industries is more evenly distributed than that of purchases.

[98] As noted earlier, multi-plant firms are considered a single operational unit, and its location corresponds to that of the firm's headquarters location.

Figure F.8: Trade Flows between Industries (2011)

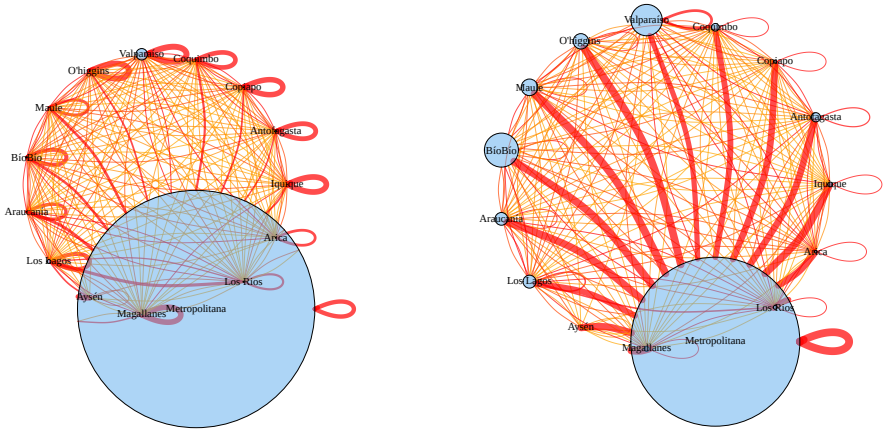


(a) Purchases (b) Sales

Notes: This figure presents the graph of the production network aggregated at the industry level. Node sizes are proportional to the industry share of purchases or sales w.r.t. to the economy. Edge width is proportional to the industry pair flow relative to all other flows from the industry of origin. Utilities are excluded from both graphs. The figures were produced with data from 2011. Nodes positions are set in circles. The location of each node in the circle is random.

Finally, network graphs in Figure F.9 summarize trade flows between regions. The structure of the graph is the same as in Figure F.8. Firms from the *Región Metropolitana*, which includes Chile's capital, Santiago, are by far the largest buyers and sellers. This is in part due to the fact that we do not have plant level information but headquarter level, which are mainly located in the capital. As noted by the width of the edges, firms tend to purchase from firms within their region, but sell to firms from the *Región Metropolitana*. This is an interesting fact. It shows that input sourcing is local whereas output selling is not.

Figure F.9: Trade Flows between Regions (2011)



(a) Purchases (b) Sales

Notes: This figure presents the graph of the production network aggregated at the regional level. Node sizes are proportional to the regional share of purchases or sales w.r.t. to the economy. Edge width is proportional to the regional pair flow relative to all other flows from the region of origin. The figures were produced with data from 2011. Nodes positions are set in circles. The location of each node in the circle is random.

F.6 Bias of Production Network Dataset Reporting Cutoff

As mentioned in the Appendix F.4 subset of firms have to report their production links. Thus, one concern is whether this introduces a bias in the sample of firms used. This concern is explored in several ways. The first is to consider the coverage of this subsample of firms. Table F.16 shows the coverage that the subsample of firms that report their complete production network represent of all firms in the economy. This table provides at least three relevant facts. The first is that although the firms that report their complete production networks are around 5 percent of the total number of firms in 2011, they represent the bulk of economic activity. The second is that the only dimension in which the coverage is relatively lower is regarding labor. This is the case since a lot of employment is concentrated in small firms. This justifies why I avoid the evaluation of the effect of propagation of shocks on the labor market. Finally, since the data started to be reported in 2003, the table shows how the coverage increased over time.[99] This implies that the quality of the data improves over time. To avoid this issue, the relevant period that the data will be used is 2006-2011 period. This year is chosen because the value added coverage stabilizes on that year onwards.

There is one final note that is relevant to mention. Although not all firms report this network information, the ones that report it might report the information of firms that don't report it.

[99] Years 2003-2004 are excluded because they are before the beginning of the other administrative datasets, and the quality of the data is worse

Table F.16: Coverage of Network Firms (%)

Variable	2005	2006	2007	2008	2009	2010	2011
Value Added	53	65	73	81	80	82	80
Sales	65	70	74	80	79	81	79
Materials	77	76	75	77	79	79	77
Investment	82	86	86	87	90	89	88
N. Employees	36	39	41	44	48	50	52
FTE Employees	37	40	42	45	49	51	52
Wage Bill	54	57	58	61	65	68	68
Exports	90	97	88	87	92	97	92
Imports	91	87	86	90	89	90	89
N. Firms	2	2	2	3	4	5	6

Notes: Share of firms' characteristics of those who report their production network relative to all firms in the economy.

Given the size threshold, this implies that transactions between large and small firms are recorded but the ones between small firms are potentially not recorded. The coverage of firms that are at least reported by other firms represents the entire economy. In other words, all firms appear one way or the other in the production networks dataset.

Despite the fact that these firms represent the majority of economic activity, there is a potential concern of whether they induce a sectorial or geographical bias for some characteristics' distribution. These concerns are addressed by Figure F.10-F.12. For both industry and geographical distribution of characteristics, one can see that the distribution of firm's characteristics is similar for the sample of all firms and the one of firms that report their production networks.

F.7 Firms' Geographic Distribution

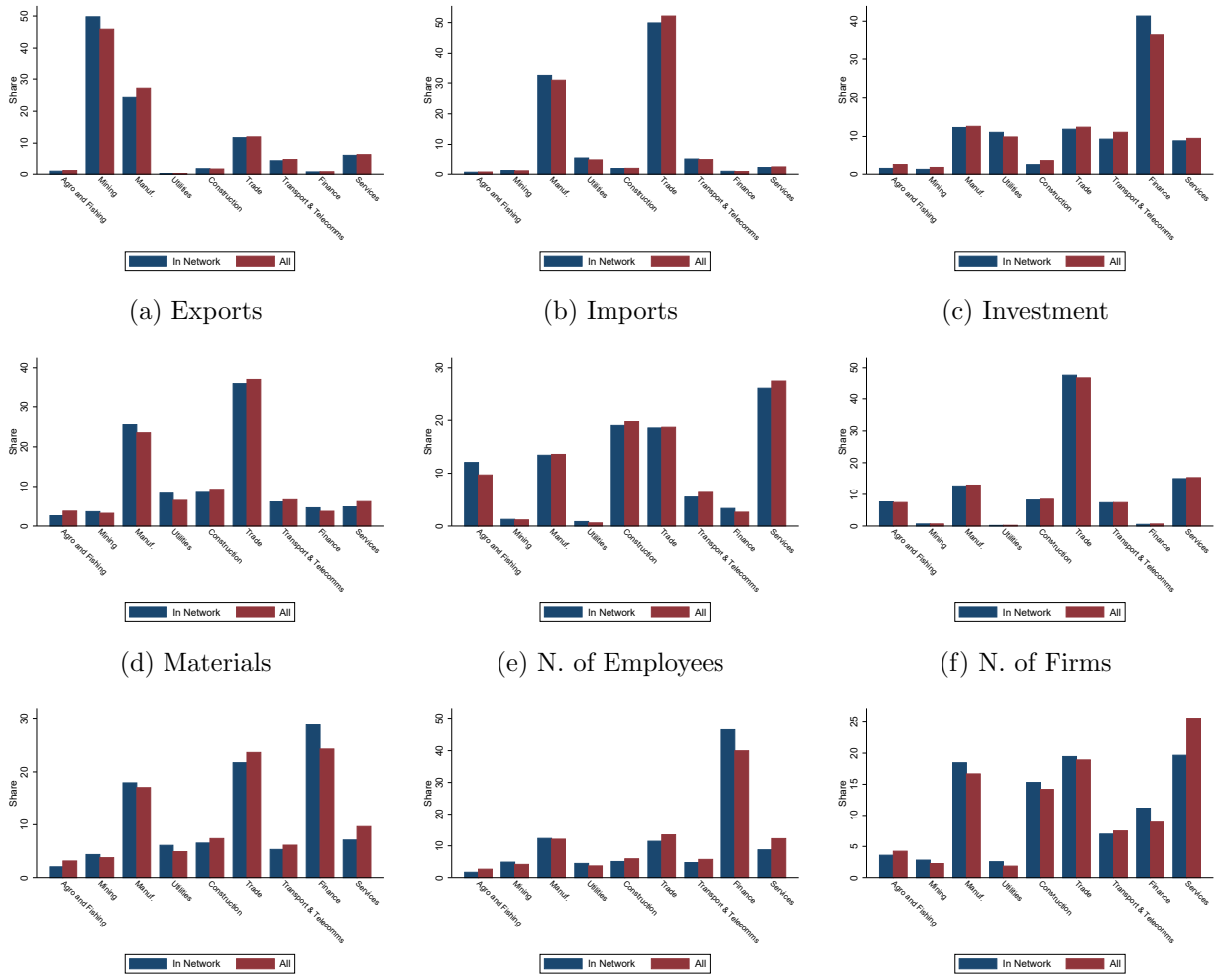
Another dimension provided by this dataset is firm's geographic location. Although I do not use this information for the analysis directly, it is used to provide validations of the dataset and further descriptive statistics. Firm's location corresponds to the place where the firm organices its tax payments, typically its headquarters. Due to confidentiality concerns, the most detailed geographic level available is *Municipalidades* (the spanish word for municipalities). Chile has 346 municipalities. Each one has a political representative called *Alcalde* (the US equivalent being the Major) that administers a public budget to provide basic services such as education and health. To identify the exact geographic location of a municipality, its downtown is located by finding the Major's office.[100] Figure F.13b has the distribution of sales of firms across these locations (the map is split between the north and the south of Chile).

F.8 Firm-to-Firm Input-Output Tables vs Industry Input-Output Tables

This appendix provides a comparison of input output tables at the industry level between official tables published by the Chilean Central Bank (CCB), and the input output tables generated from the firm pair level dataset of the SII. The section is organized as follows. First, we describe our

[100] Whenever this is not available online, an alternative public office is used such as the police.

Figure F.10: 1-Digit Industry Distribution of Firm's Characteristics - All and Reporting Network Firms - 2011



(a) Exports (b) Imports (c) Investment

(d) Materials (e) N. of Employees (f) N. of Firms

(g) Sales (h) Value Added (i) Wage Bill

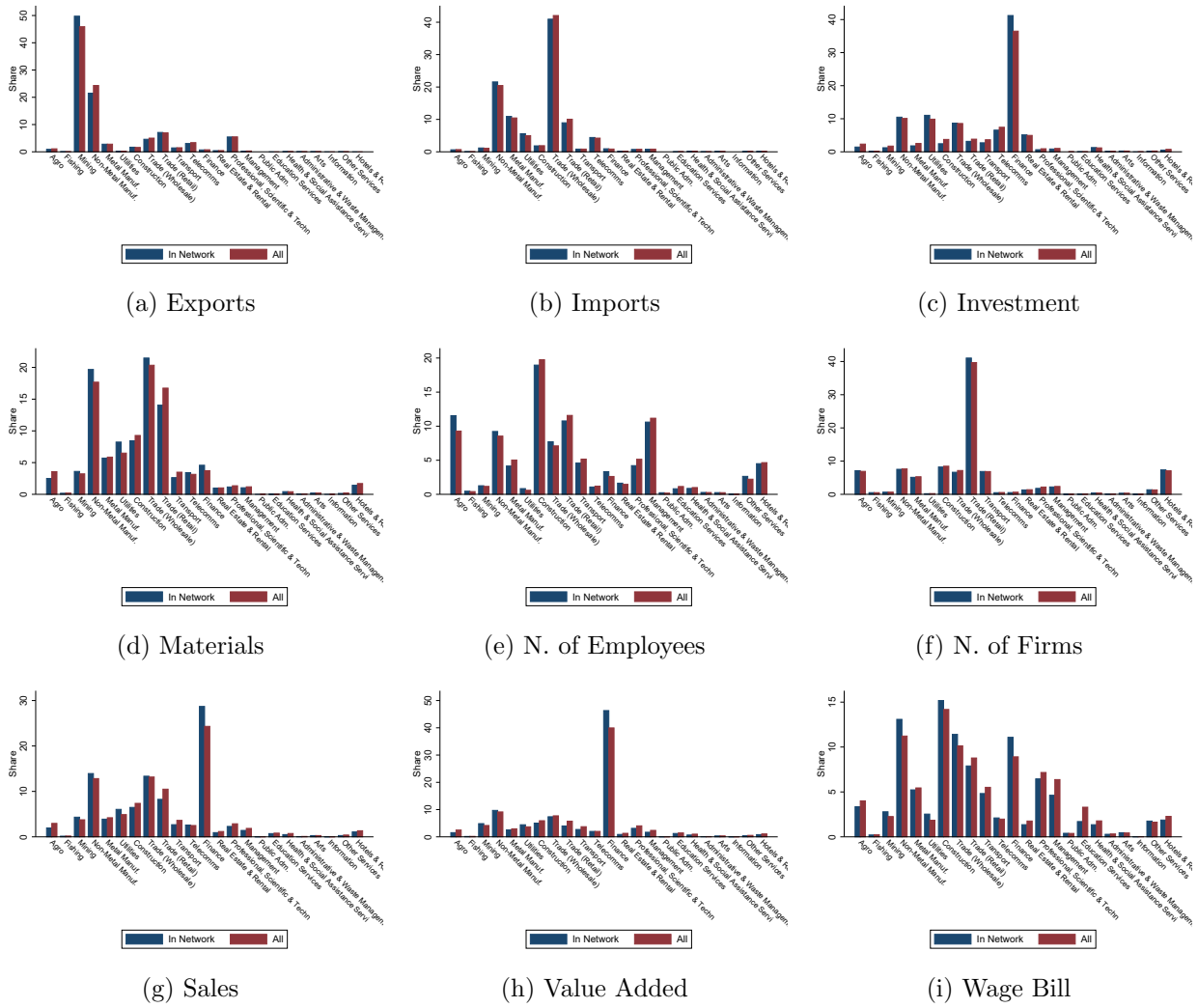
Notes: Distribution of firms' characteristics across 1-digit industry classification for the sample of all firms (All) and the one of firms that report their production networks (In Network).

method to build industry correspondences between the industry classification used by the CCB,[101] and the industry classification used by the SII. Second, we describe the results of the industry level input output tables as generated by our edge list dataset. The final step is to provide a comparison between the input-output values and coefficient between the official CCB tables and our SII tables.

Industry Classification Correspondences The input-output tables published by the CCB use a different industrial classification than that of the SII. The CCB uses a 111 sectors classification system that can be mapped to *ISIC Rev 3*. SII uses a slightly modified version of *ISIC Rev 4*. We use the official correspondence between *ISIC Rev 31* to *ISIC Rev 4* and *ISIC Rev 31* to *ISIC Rev*

[101] Central Bank's input-output tables are publicly available in their webpage, www.bcentral.cl, in particular in this link.

Figure F.11: 2-Digit Industry Distribution of Firm's Characteristics - All and Reporting Network Firms - 2011



(a) Exports (b) Imports (c) Investment

(d) Materials (e) N. of Employees (f) N. of Firms

(g) Sales (h) Value Added (i) Wage Bill

Notes: Distribution of firms' characteristics across 2-digit industry classification for the sample of all firms (All) and the one of firms that report their production networks (In Network).

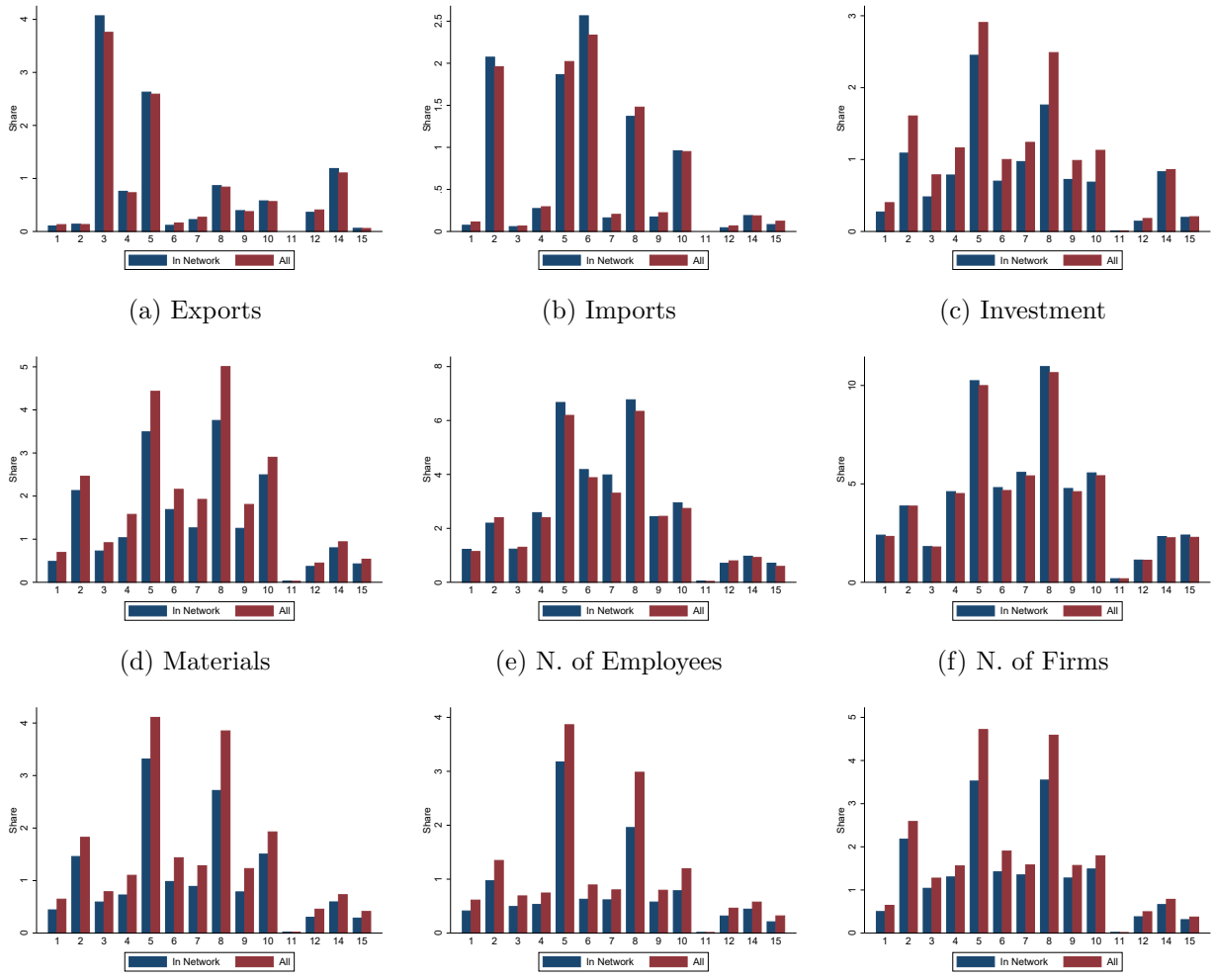
3 to link SII and CCB's industry classifications and reproduce CCB's industry level input-output tables using SII firm-to-firm level data.[102] With this procedure, we were able to generate 98 many to one or one to one correspondences between the SII industry classification and the CCB industry classification.[103] The remaining industries from the CCB classification system were linked through a many to many correspondence with the SII classification, and thus are left out of this analysis.

Firm-to-Firm Input-Output Tables I now present the distribution of input-output values,

[102]The correspondence between the CCB's industry classification and *ISIC Rev 3* was provided directly by CCB's authorities. The correspondence between the SII industries classification and *ISIC Rev 4* was provided by the Chilean National Institute of Statistics (INE). The correspondence between *ISIC* classifications was obtained from the United Nations Statistical Division, in this link.

[103]Given the industry classifications, we could merge one or many SII industries per Central Bank industry.

Figure F.12: Regional Distribution of Firm's Characteristics between Network and All Firms - 2011



(a) Exports (b) Imports (c) Investment

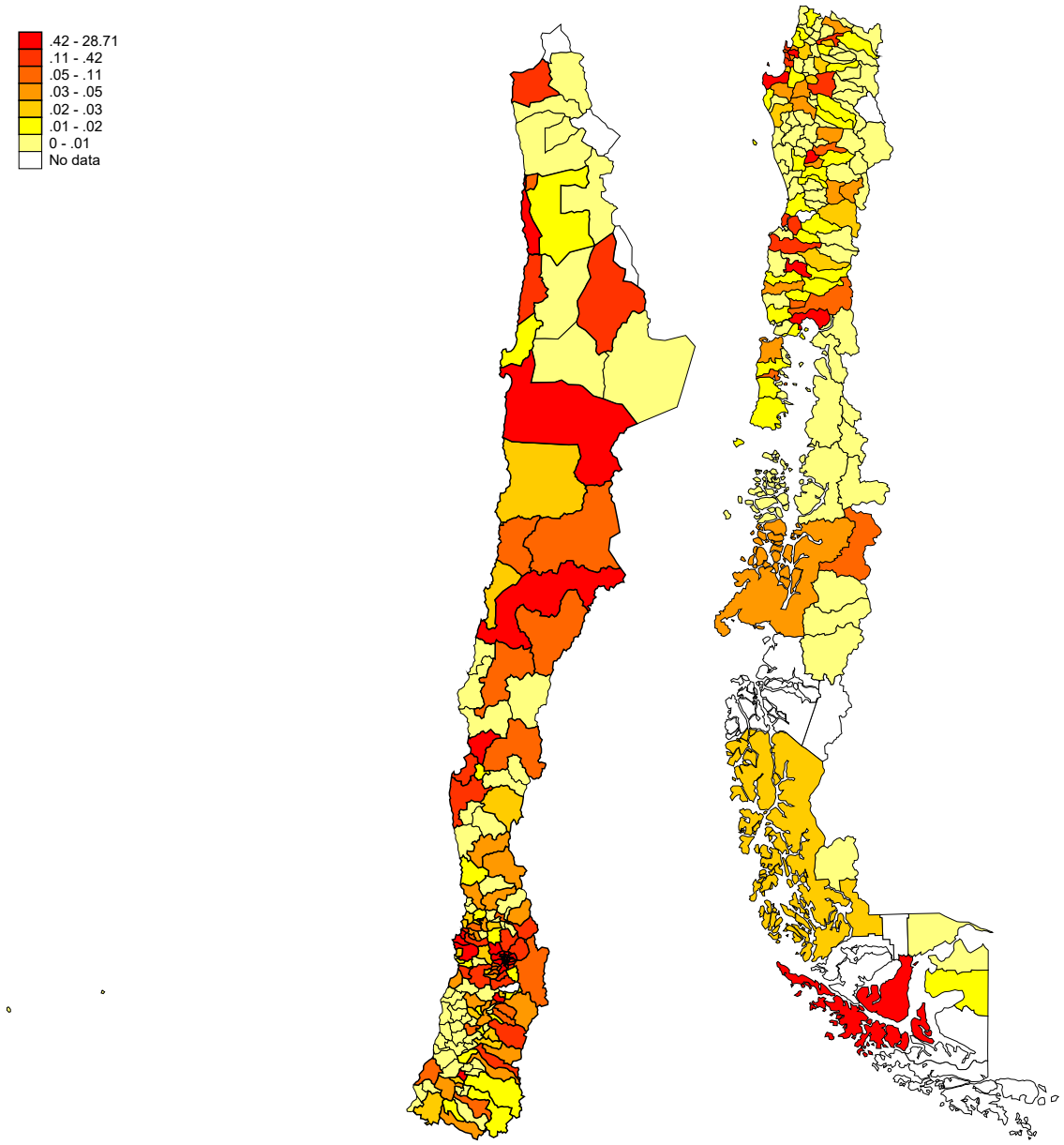
(d) Materials (e) N. of Employees (f) N. of Firms

(g) Sales (h) Value Added (i) Wage Bill

Notes: Distribution of firms' characteristics across regions for the sample of all firms (All Firms) and the one of firms that report their production networks (Reporting Networks Firms). There are 15 regions in Chile. Region 13, where Santiago (the capital) is located, is excluded because it accounts for the majority of economic activity. This is due to the fact that firm's headquarters are in general located in Santiago. And in fact, the reporting network firms are distributed relatively more in that region (for about 3 p.p. more). That is why each color does not seem to add up in this figure.

generated by the SII input-output table. The analysis is restricted to the year 2008, and to industries that were properly matched to the Central Bank's classification system, so as to provide a valid comparison for our firm-to-firm input-output table of the Chilean economy.

An input-output flow of IO_{ij} is defined as the value of the flow of goods from industry j to industry i. Direct coefficients are calculated as:



(a) Sales Distribution - North (b) Sales Distribution - South

$$\text{Direct Coefficient}_{ij} = \frac{\text{IO}_{ij}}{\text{Industry } i\text{'s Total Sales}}$$

Direct coefficients represent the share of the flow among all of the sales of the purchaser. Our data for input-output flows comes from our firm pair level dataset, and our data on industry wide sales comes from SII estimates. We further restrict the dataset to industry pairs whose direct coefficients are under the feasible boundary of 1, which leaves us with 5,310 industry-to-industry

Table F.17: SII Input Output Table (2011)

Variable	(1) Mean	(2) Standard Deviation	(3) Median
IO Values	157.693	1,201.495	1.392
Industry Sales	21.676	1.596	21.766
Direct Coefficients	0.015	0.072	0.000

Notes: IO Values and Sales in millions of 2014 US$. Direct Coefficients are a ratio.

linkages.

Figure F.14 and Table F.17 summarize the input-output values, industry level sales, and direct coefficients we observe from our firm level table. The distribution of input output values is highly skewed, the mean is 157 millions of US$ and the standard deviation is 1,201 millions of US$. The median value is much lower, at 1.3 million of US$. Most of input-output values are in the range between 0 and 100 million US$. Direct Coefficients are also highly skewed, with a mean of 0.015 and a standard deviation of 0.072. The median value for direct coefficients is 0.000. These moments suggest that most industries have non intensive pairwise relations. Industry level sales have a less skewed distribution, the average industry sells 6,239 millions of US$, with a standard deviation of 7,787 US$ and a median value of 2,836 US$.

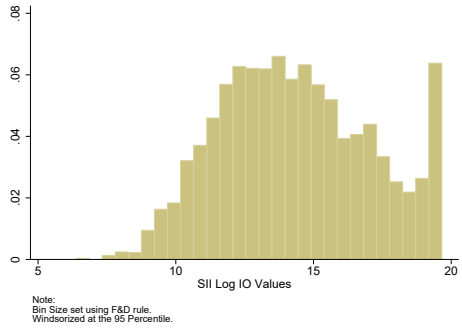
Input-Output Tables Comparison I provide a comparison of the input-output values and direct coefficients between the official Central Bank table and our firm level table. As the scatter plots in Figures F.15b and F.15a show, input-output values and industry sales have a positive though not perfectly linear relation between sources. Also, according to the histograms in Figures F.16a, F.16c and F.16b, the differences are centered around zero.

Table F.18 provides descriptive statistics for the differences between input-output flows and direct coefficients at both sources, were we subtracted CCB's input-output values and direct coefficients to our SII calculations. The mean of the differences between Input-Output values is of 24 millions of US$, and the median is just 0.068 millions of US$. However, there is high dispersion in these values, as the standard deviation of the differences is 81 millions of US$. This implies that our SII input-output flows are, on average, larger than those calculated by the CCB. The mean of the differences between direct coefficients is 0.01, and the median nears zero. The standard deviation of this values is of 0.073. The direct coefficients, as calculated by our SII input-output table are, on average, higher than those of the calculations by the CCB. Industry level sales are also higher at SII data, the mean difference between SII and CCB values is 1,941 US$, while the median is 1,868 US$. It's standard deviation is 4,068 US$. A more careful and detailed comparison of input-output tables between these two sources of data is left for future research.

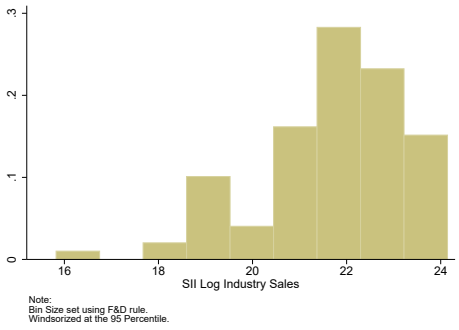
F.9 Merging Chilean Customs Data with International Trade Data

Customs This data appendix describes the merge between Chile's Customs data and international data of global flows and prices between different countries for a detail list of products. The Chilean

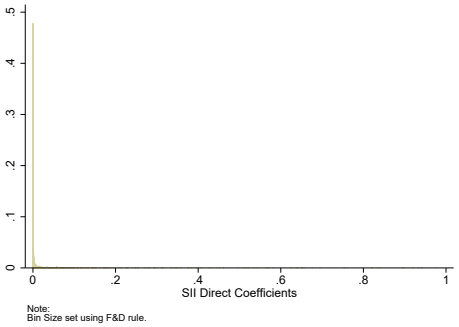
Figure F.14: SII Input Output Table (2011)



(a) IO Values



(b) Industry Sales



(c) Direct Coefficients

Notes: This figure presents the distribution of input output values and direct coefficients across industry pairs using the Central Bank industry classification system. Bin sizes are set using the Freedman & Diaconis rule. The figures were produced with SII data from 2011.

Table F.18: Input-Output Table Differences (2011).

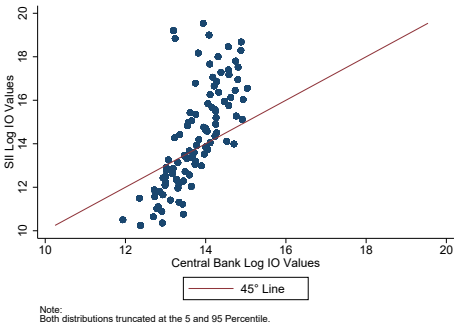
Difference between SII and Central Bank	(1) Mean	(2) Standard Deviation	(3) Median
IO Values	24.805	81.417	0.068
Industry Sales	1,941.784	4,068.841	1,868.738
Direct Coefficients	0.010	0.073	0.000

Notes: IO Values and Sales in millions of 2014 US$. Direct Coefficients are a ratio.

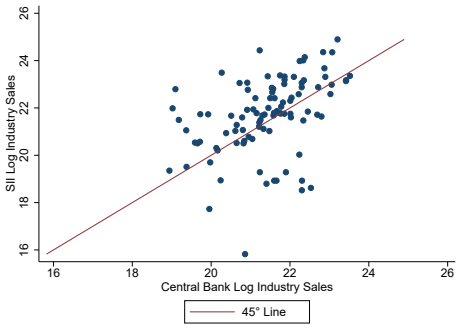
Customs Authorities (*Servicio Nacional de Aduanas*), through the Secretary of Treasury, provided imports and exports transactions data between the years 2002 and 2015. The dataset contains information on the transaction values (free on board (FOB) for exports and cost, insurance and freight (CIF) for imports), the quantity of goods associated to the transaction, the product code according to the 6 Digits Harmonized Commodity Description and Coding System (henceforth *HS Code*), and the country of destination of exports and country of origin of imports.[104]

[104]The countries are classified according to the Chilean Customs Classification System, available in this link.

Figure F.15: Scatter Plots of Input Output Values and Direct Coefficients (2011)



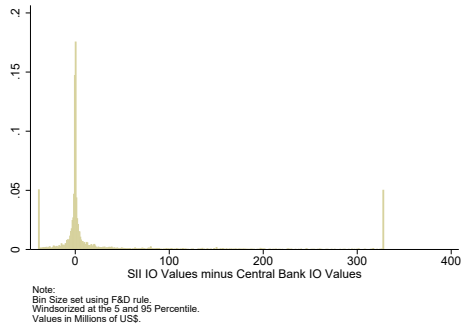
(a) IO Values



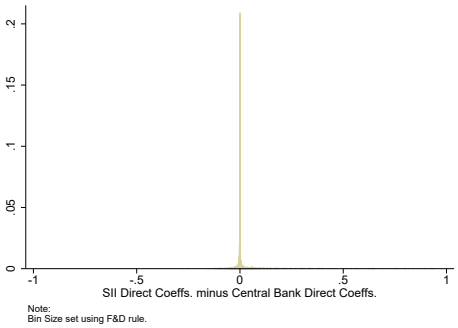
(b) Direct Coefficients

Notes: This figures present the linear relation of input output values and direct coefficients between SII data and Central Bank data. The unit of analysis are industry pairs according to the Central Bank industry classification system. The figures were produced with SII and CCB data from 2011.

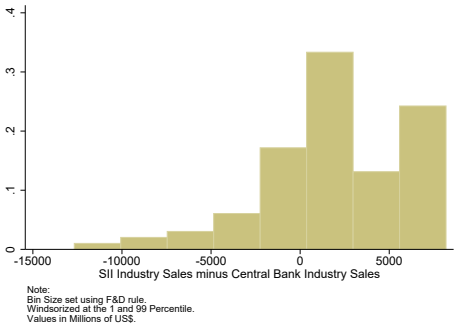
Figure F.16: Histograms of Differences



(a) IO Values



(b) Direct Coefficients



(c) Industry Sales

Notes: This figures present the distribution of differences for input output values and direct coefficients between SII data and Central Bank data. The unit of analysis are industry pairs according to the Central Bank industry classification system. Bin sizes are set using the Freedman & Diaconis rule. The figures were produced with SII and CCB data from 2011.

BACI The customs data is merged with international data between countries for a detail list of products. This data is called BACI and is published by *CEPII*,[105,106] and its original data comes from the UN's Comtrade Dataset.[107] It contains data on country-product level bilateral values and quantities of exports between the years 2003 and 2015, products are classified by their 6 Digit *HS Codes* according to revisions *HS 2002*, *HS 2007* and *HS 2012*. Country Codes are classified using their official UN codes.[108]

F.9.1 Merge Procedure

Country Code Correspondences The merge between the customs and the BACI dataset is implemented at the country-product combination. In order to do this, both country and product identification are standardized between the two datasets. I produced the country codes correspondence between Customs and Comtrade manually by imputing the *ISO 2*[109] country code to the list of countries at Customs dataset.[110] Most countries have a one to one relation between the two sources. Exceptions are mainly due to Comtrade qualifying trade flows from small countries as if they were from a large neighboring country, or because offshore dependencies are qualified as separate entities in the Customs classification.[111]

***HS Codes* Correspondences** Using the official tables[112], we produced the product code correspondences between revisions *HS 2002*, *HS 2007* and *HS 2012*. Although we used data from revision *HS 2012* as our baseline, we also used, by means of the correspondences tables between *HS Codes* revisions, information based on revisions *HS 2002* and *2007*.

Merge Level Our baseline dataset are yearly country-product firm level exports and imports from Customs, for which we have the countries of origin or destination plus the traded products 6 digit *HS Code*. We successively match this dataset with yearly BACI country-product level data under product *HS Code* revisions *HS 2002*, *HS 2007* and *HS 2012*. We set the final product code to match that of the latests *HS Code* revision available for the product-year pair. This means that if a product is available at revisions *HS 2002*, *HS 2007* and *HS 2012*, then its product code reflects that of the *HS 2012* classification system, and its corresponding CIF or FOB values and quantities from BACI's information. In case a recent HS version is not available at any given moment, then the previous HS version is used to do the merge. This type of cases can occur, for example, because in transition years between HS versions, firms can report using the old version rather than the new. Nevertheless, as is shown in the next section, most of trade flows are covered by the latest *HS Code*

[105] The French *Centre d'Etudes Prospectives et d'Informations Internationales*.
[106] For the official presentation of the BACI dataset, visit this link.
[107] The Comtrade Dataset is published by the United Nations International Trade Statistics Database. This dataset summarizes import and export trade flows at the country and product level. For a complete description of the features of the Comtrade dataset, check their official presentation.
[108] Available in this link.
[109] The official classification is available in this link
[110] The dataset is available in this link.
[111] For example, trade flows from Liechstenstein are qualified by Comtrade as from Switzerland. Also, Puerto Rico, the Virgin Islands and the US Pacific Ocean Territories are qualified by Customs separately from the US.
[112] Available in this link

Table F.19: Number of Exported Products

Year	(1) HS 2002	(2) HS 2007	(3) HS 2012	(4) Customs Only	(5) BACI Only
2004	220	98	3,139	8	2,345
2005	216	92	3,166	6	2,328
2006	202	85	3,136	7	2,379
2007	0	92	3,323	10	2,386
2008	0	95	3,285	8	2,421
2009	0	95	3,148	8	2,557
2010	0	95	3,103	13	2,596
2011	0	91	3,180	10	2,507
2012	0	0	3,317	11	2,467
2013	0	0	3,228	12	2,550
2014	0	0	3,247	12	2,519

Notes: Columns (1) to (3) are number of matched products at each revision. Columns (4) and (5) are unmatched products.

revision.

F.9.2 Descriptive Statistics

This section describes the availability of products and country product pairs at both the Customs and BACI datasets, and the quality of the merge between them. First we check the number of product codes available at each datasets, and the share of trade flows accounted by the merge between them at the product level. Then we check the number of country product pairs available at both datasets, and the share of trade flows accounted by the match between them at the country-product pairs level.

Product Level Merge Tables F.19 and F.20 report the number of product codes, plus their *HS Code* revision, that were available at both the Customs and BACI datasets, and the number of product codes that were available only at the Customs or BACI datasets. As our matching algorithm prioritizes later *HS Code* revisions, most of matched product codes correspond to the *HS 2012* revision, which represent by far the largest fraction of matched product codes between years 2004 and 2011, and all of matched product codes after 2011. *HS 2002* covers a small fraction of products between years 2004 and 2006, and *HS 2007* covers an even smaller fraction of products between years 2004 and 2011. Product codes available only at the Customs datasets are very few, for both exports and imports. Product codes available only at the BACI dataset are in the order of 2,000's for exports and 1,000's for imports, relative to a total of approximately 5 thousand products.

The next step is to analyze the share of trade flows at each source that are covered by the matched sample of products. The following calculation is performed:

Table F.20: Number of Imported Products

Year	(1) HS 2002	(2) HS 2007	(3) HS 2012	(4) Customs Only	(5) BACI Only
2004	369	128	4,175	36	1,130
2005	368	121	4,180	40	1,133
2006	368	120	4,184	35	113
2007	0	131	4,451	40	1,219
2008	0	135	4,456	36	1,210
2009	0	126	4,457	36	1,217
2010	0	135	4,490	35	1,169
2011	0	142	4,479	36	1,157
2012	0	0	4,689	34	1,095
2013	0	0	4,665	32	1,113
2014	0	0	4,664	32	1,102

Notes: Columns (1) to (3) are number of matched products at each revision. Columns (4) and (5) are unmatched products.

$$\text{Exports Share}_j = \frac{\text{Matched Products Export Flows}_j}{\text{All Export Flows}_j}$$

$$\text{Imports Share}_j = \frac{\text{Matched Products Import Flows}_j}{\text{All Import Flows}_j}$$

where j is either BACI or Customs. The results are shown in Table F.21. The share of export trade flows from the Customs datasets that is merged with the BACI dataset is high (around 95 percent of export flows). Similarly, the share of import trade flows from the Customs dataset that is merged with the BACI dataset is also high, but lower than that of exports (around 90 percent of total import flows). From the BACI dataset's point of view, the share of trade flows merged with Chilean customs is significantly lower. Between 60 to 70 percent of exports and 70 to 80 percent of imports flows from the BACI dataset are merged with the Customs dataset. This does not come as a surprise, since there are many international products that Chilean firms neither export nor import.

Country-Product Level Merge Tables F.19 and F.20 report the number of product codes and country codes pairs, plus their *HS Code* revision, that were available at both the Customs and BACI datasets, and the number of product code and country code pairs that were available only at the Customs or BACI datasets. Again, as our matching algorithm prioritizes later *HS Code* revisions, most of matched product codes correspond to the *HS 2012* revision, which represent the largest fraction of matched product codes between years 2004 and 2014. *HS 2002* covers a small fraction of products between years 2004 and 2006 and year 2014, and *HS 2007* covers a small fraction of products between years 2004 and 2014. Product code and country code pairs available only at the Customs datasets are very few relative to the number of matched pairs, for both exports and imports. Product codes and country codes pairs available only at the BACI dataset are much higher than that of the matched pairs, in the order of 800 thousands for exports and 400 thousands

Table F.21: Matched Products Trade Coverage

Year	(1) Customs Exports	(2) Customs Imports	(3) BACI Exports	(4) BACI Imports
2004	96.48	96.00	60.11	70.90
2005	95.74	94.24	59.30	70.31
2006	96.09	92.75	58.81	70.67
2007	96.61	87.70	66.85	81.33
2008	95.37	86.70	63.39	79.30
2009	96.17	90.24	64.15	79.24
2010	97.07	90.09	62.71	79.22
2011	96.49	90.39	61.61	79.31
2012	96.60	89.70	66.21	81.46
2013	96.78	91.49	66.73	83.81
2014	96.69	91.31	67.60	83.22

Notes: Columns (1) to (4) represent the share of trade flows covered by matched products.

Table F.22: Number of Exports Country-Products Pairs

Year	(1) HS 2002	(2) HS 2007	(3) HS 2012	(4) Customs Only	(5) BACI Only
2004	1,535	767	20,213	293	783,339
2005	1,548	789	20,505	268	809,671
2006	1,520	795	21,345	235	813,913
2007	0	1,013	23,307	233	812,322
2008	0	1,074	23,053	228	807,516
2009	0	1,052	21,721	223	799,730
2010	0	1,001	21,152	195	805,041
2011	0	1,015	21,957	255	809,890
2012	0	6	22,674	333	783,701
2013	0	1	22,397	331	787,741
2014	1	1	23,014	321	786,146

Notes: Columns (1) to (3) are number of matched country-products at each revision. Columns (4) and (5) are unmatched country-products.

for imports.

In Table F.24 we check the coverage of trade flows accounted by matched product and country codes from each dataset. Specifically, we perform the following calculations:

$$\text{Exports Share}_j = \frac{\text{Matched Country-Products Export Flows}_j}{\text{All Export Flows}_j}$$

$$\text{Imports Share}_j = \frac{\text{Matched Country-Products Import Flows}_j}{\text{All Import Flows}_j}$$

Were j is either BACI or Customs. Most of trade flows from the customs dataset are properly matched with the BACI dataset. Around 90 percent of exports and 95 percent of imports flows from the Customs dataset were matched with the BACI dataset. This changes for the BACI datasets,

Table F.23: Number of Imports Country-Products Pairs

Year	(1) HS 2002	(2) HS 2007	(3) HS 2012	(4) Customs Only	(5) BACI Only
2004	3,452	727	46,836	3,581	425,819
2005	3,430	699	47,914	3,708	444,610
2006	3,567	696	49,534	3,777	450,307
2007	0	845	53,135	3,959	453,562
2008	0	958	57,611	4,340	448,259
2009	1	928	56,065	4,653	441,353
2010	0	993	58,783	3,994	439,580
2011	0	1,067	60,671	3,429	438,819
2012	19	93	63,565	3,605	416,669
2013	20	44	64,565	3,487	412,528
2014	2	24	64,494	3,857	408,222

Notes: Columns (1) to (3) are number of matched country-products at each revision. Columns (4) and (5) are unmatched country-products.

Table F.24: Matched Country-Product Pairs Trade Coverage

Year	(1) Customs Exports	(2) Customs Imports	(3) BACI Exports	(4) BACI Imports
2004	94.90	96.47	19.13	47.96
2005	93.50	95.67	19.31	47.13
2006	91.77	96.05	18.31	47.12
2007	87.04	96.55	20.48	52.63
2008	85.99	95.37	18.04	49.74
2009	89.32	96.06	18.45	51.61
2010	89.32	96.97	18.11	51.04
2011	89.59	96.45	17.42	50.32
2012	88.83	96.57	20.37	55.38
2013	90.46	96.75	20.90	55.68
2014	90.28	96.67	21.43	57.39

Notes: Columns (1) to (4) represent the share of trade flows covered by matched country-product pairs.

as the product and country code pairs merged with Customs dataset are around 20 percent for exports and 50 percent for imports. The lower coverage of product code and country product code is due to the fact that Chile does only trade in a subset of all active product and country code pairs in world trade, for which the BACI dataset is a good proxy.

Overall, Tables F.21 and F.24 show that the merge between the Customs and BACI dataset covers the majority of international trade activity performed by firms from Chile, either through exports or imports.